University of Colorado Boulder

Math 2400, Final Exam

Spring 2017

PRINT YOUR NAME: ______________________________

PRINT INSTRUCTOR'S NAME: ______________________________

SECTION #: ______________________________

Question	Points	Score
1	16	
2	18	
3	5	
4	5	
5	8	
6	12	
7	12	
8	12	
9	12	
Total:	100	

- No calculators, cell phones, or other electronic devices may be used at any time during the exam.
- Show all of your reasoning and work for full credit, unless indicated otherwise. Use full mathematical or English sentences.
- You have 150 minutes and the exam is 100 points.
- You do not need to simplify numerical expressions. For example leave fractions like 100/7 or expressions like ln(3)/2 as is.
- For multiple choice questions, circle the correct answer.
- Notation: following the book, we use boldface to denote vectors, e.g., $\mathbf{a}, \mathbf{b}$ are vectors.

1. (16 points) You do not have to justify your answers in this question.

   (a) (2 pts.) True/False: Two vectors $\mathbf{a}$ and $\mathbf{b}$ are orthogonal if and only if $\mathbf{a} \cdot \mathbf{b} = 0$.

   (b) (2 pts.) Complete the statement: If a curve C is parametrized with respect to arc length, then

   $$\int_0^t |\mathbf{r}'(u)| du =$$

   (c) (4pts.) Use spherical coordinates to set up the triple integral for the volume of the unit sphere. Do not evaluate.

(d) (2 pts.) Let C be a unit circle oriented counterclockwise, centered at $(0,0)$, and let $f(x,y) = \cos x + \ln(2-y)$. Evaluate

$$\int_C \nabla f \cdot d\mathbf{r} =$$

(e) (2 pts.) True/False: Let f be a scalar valued function, then the following expression is meaningful:

$$\operatorname{curl} \operatorname{div} f.$$

(f) (2 pts.) Complete the statement: Let $\mathbf{F}$ be a conservative vector field on $\mathbb{R}^3$, then

$\operatorname{curl} \mathbf{F} =$

(Do not give the definition of curl.)

(g) (2 pts.) Finish the formula for the Green's Theorem: Let $\mathbf{F}(x,y) = \langle P(x,y), Q(x,y) \rangle$, if D is a region bounded by C, where C is a piecewise-smooth, positively oriented, simple closed curve in the plane, then

$$\int_C P\ dx + Q\ dy =$$

2. (18 points) Short answer questions: show work here and in the rest of the exam.

   (a) (4 pts.) Find an equation of a plane that contains a point $(2, 3, 4)$ and is perpendicular to the vector $\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}$.

   (b) (4 pts.) Let $f(x, y)$ be given, where $x = u + v$ and $y = u - v$. Use the Chain Rule to compute

   $$\frac{\partial f}{\partial v} =$$

(c) (6 pts.) Use Lagrange multipliers to set up a system of four equations to find the shortest distance from the point $(2, 0, -3)$ to the plane $x + y + z = 1$. Do not solve for x, y, z, λ.

(d) (4 pts.) Compute the Jacobian of the change of variables (x, y) to polar coordinates (r, θ):

$$x = r\cos\theta,\ \ y = r\sin\theta.$$

Show work.

3. (5 points) Find a vector equation of the tangent line to the helix with parametric equations

$$x(t) = \cos t, \quad y(t) = \sin t, \quad z(t) = t,$$

at the point $(0, 1, \frac{\pi}{2})$.

4. (5 points) Demonstrate that the following limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{y^2}{x^2+y^2}$$

5. (8 points) Suppose you are climbing a hill whose shape is given by $z = 6 - x^2 - y^2$, and you are standing at the point $(1, 2, 1)$.

   (a) (3 pts.) In which direction should you walk to go up with the most difficulty?

   (b) (2 pts.) What is the rate of ascent in that direction?

   (c) (3 pts.) You are still at the point $(1, 2, 1)$. If you walk in the direction of the vector $\mathbf{i} + \sqrt{3}\mathbf{j}$, will you ascend or descend? With what rate?

6. (12 points) Let $f(x, y) = 3x^2 + 3x^2y + 6y^2$. Find all local minima, maxima and saddle points if any. Give both location and value for each.

7. (12 points) Evaluate the surface integral

$$\iint_S x \, dS,$$

where S is the part of the cone $z = \sqrt{x^2 + y^2}$ in the first octant that lies between the plane $y = x$ and the parabolic cylinder $y = x^2$.

8. (12 points) Use Stokes' Theorem to show the surface integral $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S} = 0$ where

$$\mathbf{F}(x, y, z) = \langle yz, xy, xz \rangle,$$

and S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $y^2 + z^2 = 1$ with $x \geq 0$.

9. (12 points) Use the Divergence Theorem to calculate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where

$$\mathbf{F}(x, y, z) = \langle z^3, (\pi - x)^4, \frac{z}{9\sqrt{x^2 + y^2}} \rangle,$$

and S is the positively (outward) oriented boundary of E, where E is the solid in the first octant beneath the paraboloid $z = x^2 + y^2$ and inside the cylinder $x^2 + y^2 = 9$.