University of Colorado Boulder

Math 2400, Midterm 3

Spring 2017

PRINT YOUR NAME: ______________________________

PRINT INSTRUCTOR'S NAME: ______________________________

SECTION #: ______________________________

Question	Points	Score
1	13	
2	16	
3	8	
4	8	
5	15	
6	8	
7	16	
8	16	
Total:	100	

- No calculators, cell phones, or other electronic devices may be used at any time during the exam.
- Show all of your reasoning and work for full credit, unless indicated otherwise. Use full mathematical or English sentences.
- You have 90 minutes and the exam is 100 points.
- You do not need to simplify numerical expressions. For example leave fractions like $100/7$ or expressions like $\ln(3)/2$ as is.
- For multiple choice questions, circle the correct answer.
- Notation: following the book, we use boldface to denote vectors, e.g., $\mathbf{a}, \mathbf{b}$ are vectors.

1. (13 points) Short answer questions.

   (a) (5 pts.) Let $R = [0,2] \times [0,2]$. Use a Riemann sum with $m = n = 2$ to estimate the value of $\iint\limits_R x + y \, dA$. Take the sample points to be the lower left corners.

   (b) (3 pts.) Let the density function of a lamina D be $\rho(x,y) = x^2 y$. Use a double integral to write a formula for the mass of the lamina D.

   (c) (5 pts.) Let $\mathbf{F} = \langle x, y, z \rangle$, and C be a curve parameterized by $\mathbf{r}(t) = \langle t, e^t, \sin t \rangle$, $0 \le t \le 1$. Set up, but DO NOT evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$.

$$\int_{\square}^{\square} \boxed{\qquad\qquad} \cdot \boxed{\qquad\qquad} \, d\boxed{\quad}$$

$$= \int_{\square}^{\square} \boxed{\qquad} + \boxed{\qquad\qquad} + \boxed{\qquad\qquad} \, d\boxed{\quad}$$

2. (16 points) (a) (8 pts.) Evaluate the double integral,

$$\iint_D x \; dA,$$

where,

$$D = \{(x, y) : 0 \leq x \leq 1,\ 2x^2 \leq y \leq 1 + x^2\}.$$

(b) (8 pts.) Evaluate the following double integral by changing the order of integration.

$$\int_0^1 \int_{2x}^2 e^{y^2} \, dy dx =$$

3. (8 points) Evaluate the triple intergral

$$\iiint_E y \, dV,$$

where E is the solid in the first octant bounded by the parabolic cylinder $y = x^2$ and the planes $z = 2$, and $y = 1$.

4. (8 points) Using spherical coordinates, set up an integral which finds the volume of the solid that lies within the sphere $x^2 + y^2 + z^2 = 25$, above the xy-plane, and below the cone $z = \sqrt{x^2 + y^2}$. DO NOT evaluate.

$$\int_{\square}^{\square} \int_{\square}^{\square} \int_{\square}^{\square} \boxed{} \, d\boxed{} \, d\boxed{} \, d\boxed{}$$

5. (15 points) Let $a, b > 0$. In the parts that follow, you will show that the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1$ has area πab by making the following change of variables

$$x = ar\cos\theta, \quad y = br\sin\theta.$$

(a) (5 pts.) Compute the Jacobian of the transformation.

$$\frac{\partial(x,y)}{\partial(r,\theta)} =$$

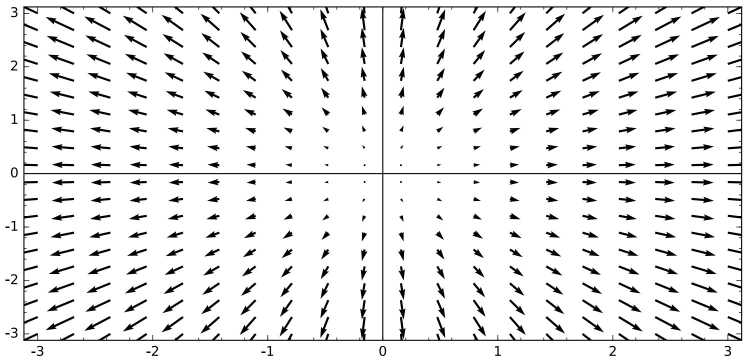
(b) (5 pts.) Under this change of variables, the ellipse is the image of a disk. Find the radius of that disk. (Hint: plug in the change of variables to the equation for the ellipse.)

(c) (5 pts.) Using parts (a) and (b), apply the change of variables formula for a double integral to compute the area of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1.$$

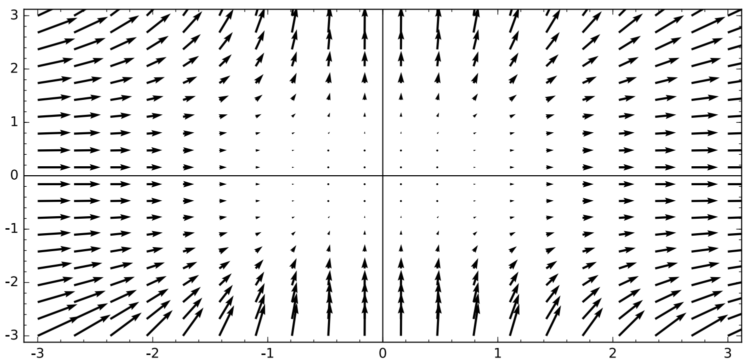
6. (8 points) Match the vector field with the equation describing the vector field. No justification is needed.

1. Vector Field 1: __________



I.) $\mathbf{F}(x, y) = (2-x)\mathbf{i}+(3-y)\mathbf{j}$

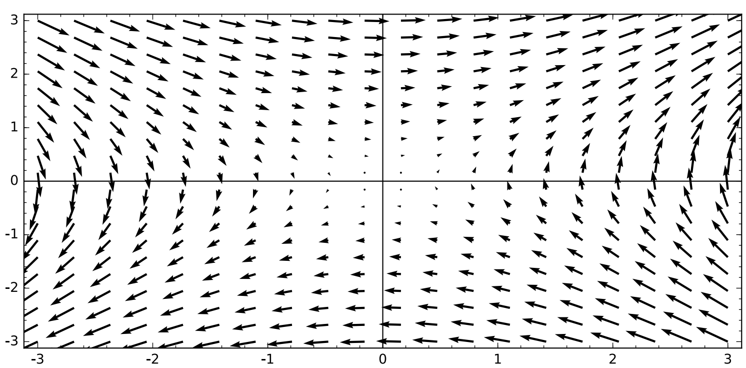
2. Vector Field 2: __________



II.) $\mathbf{F}(x, y) = y\mathbf{i} + x\mathbf{j}$

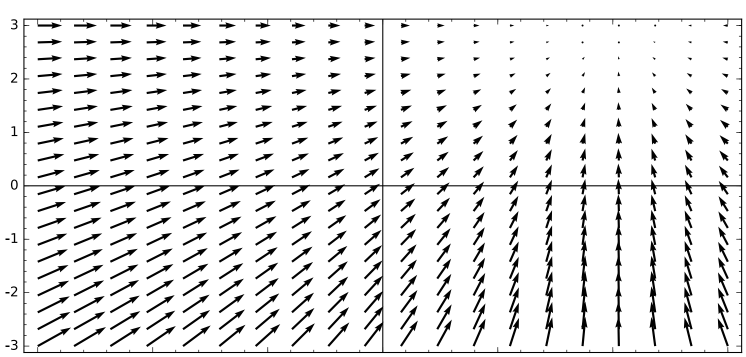
III.) $\mathbf{F}(x, y) = x^2\mathbf{i} + y^2\mathbf{j}$

3. Vector Field 3: __________



IV.) $\mathbf{F}(x, y) = x\mathbf{i} + y\mathbf{j}$

4. Vector Field 4: __________



7. (16 points) Let C be the curve traced out by traversing the upper half of a circle of radius 3 counterclockwise from $(3, 0)$ and then traveling along the line segment from the point $(-3, 0)$ to the point $(3, 0)$.

   (a) (8 pts.) Find a parametrization for C. You may divide C into multiple segments.

   (b) (8 pts.) Calculate $\int_C y\, ds$.

8. (16 points) Let S be the surface of the solid obtained by taking a section of the cylinder $x^2 + y^2 = 1$ between the planes $z = 2 - y$ and $z = 0$.

(a) (6 pts.) The upper face of S, the part lying in the plane $z = 2 - y$, may be parametrized by $\mathbf{r}(x, y) = \langle x, y, 2 - y \rangle$, where $(x, y) \in \{(x, y) : x^2 + y^2 \leq 1\}$. Compute the surface area of that portion of the surface S.

(b) (6 pts.) The portion of the surface S lying on the cylinder $x^2 + y^2 = 1$ may be parametrized by $\mathbf{r}(\theta, z) = \langle \cos(\theta), \sin(\theta), z \rangle$. Find the bounds for θ and z and then calculate the surface area of that portion of S.

(c) (4 pts.) What is the total surface area of S?