

# MATH 2400: CALCULUS 3

5:15 - 6:45 pm, Mon. Apr. 11, 2016

## MIDTERM 3

I have neither given nor received aid on this exam.

Name: \_\_\_\_\_

Check one below !

- |   |   |
|---|---|
| <input type="radio"/> <b>001</b> WATTS .....(9AM)       | <input type="radio"/> <b>005</b> WASHABAUGH ..... (1PM) |
| <input type="radio"/> <b>002</b> GREEN ..... (10AM)     | <input type="radio"/> <b>006</b> BULIN ..... (2PM)      |
| <input type="radio"/> <b>003</b> BLAKESTAD ..... (11AM) | <input type="radio"/> <b>007</b> CHHAY .....(3PM)       |
| <input type="radio"/> <b>004</b> MISHEV .....(12PM)     |   |

Notes, electronic devices, and any other aids are **not** permitted on this exam.

If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete, logical, legible, and correct**. Show all of your work, and give adequate explanations. No shown work even with the correct final answer will lead to no points. Only give one answer to each problem! If there are two different answers to one problem, the lower score will be chosen.

**DO NOT WRITE IN THIS BOX!**

<b>Problem</b>	<b>Points</b>	<b>Score</b>
<b>1</b>	16 pts	
<b>2</b>	17 pts	
<b>3</b>	17 pts	
<b>4</b>	17 pts	
<b>5</b>	16 pts	
<b>6</b>	17 pts	
<b>TOTAL</b>	100 pts	

1. (16 points) Evaluate the integral

$$\iiint_R y \, dV,$$

where  $R$  is the finite region bounded by  $z = 0$ ,  $x = 0$ ,  $z = 1 - y$  and  $y = \sqrt{x}$ .

2. (17 points) Suppose that a thin wire is bent into the shape of the semicircle  $x^2 + y^2 = 4$ ,  $y \geq 0$ . Suppose moreover that the density of the wire is given by  $\rho(x, y) = 3 + x$ . Find the mass and center of mass of the wire.

3. (17 points) Compute the surface area of the part of the cylinder  $x^2 + y^2 = 1$  that lies between the planes  $z = 0$  and  $x + y + z = 10$ .

4. (17 points) Let  $\mathbf{F}(x, y) = \langle 3y^2 + 4xe^y, 6xy + 2x^2e^y \rangle$ .

(a) (7 points) Is  $\mathbf{F}$  conservative? If yes, find a potential function  $f$  for it. If not, justify your answer.

(b) (10 points) Let  $C$  be the curve starting at  $(0, 0)$ , following the ellipse  $(x - 1)^2 + \frac{1}{4}y^2 = 1$  counterclockwise and ending at  $(2, 0)$ . Compute the line integral  $\int_C \mathbf{F} \cdot d\mathbf{r}$ .

5. (16 points) Let  $C$  be the closed curve in the  $xy$ -plane consisting of the right half of the circle of radius 2 centered at the origin and the line segment from  $(0, 2)$  to  $(0, -2)$ . Suppose  $C$  is positively oriented (i.e.  $C$  is traversed counterclockwise).

(a) (8 points) Evaluate the line integral

$$\int_C y \, dx + (y - x) \, dy$$

directly by parameterizing the curve  $C$ .

(b) (8 points) Evaluate the same line integral

$$\int_C y \, dx + (y - x) \, dy$$

by using Green's Theorem.

6. (17 points) Evaluate

$$\iint_R \sqrt{xy^3} \, dA,$$

where  $R$  is the region in the  $xy$ -plane bounded by  $xy = 1$ ,  $xy = 9$ ,  $y = x$  and  $y = 4x$ .

**Hint:** Use the substitution  $u = xy$ ,  $v = \frac{y}{x}$ .