

# CALCULUS 3

October 13, 2010

## 2nd TEST

**YOUR NAME:**

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| <input type="radio"/> <b>002</b> J. HARPER ..... (9AM)<br><input type="radio"/> <b>003</b> C. MESA ..... (10AM)<br><input type="radio"/> <b>004</b> A. SPINA ..... (11AM) | <input type="radio"/> <b>005</b> A. SPINA ..... (12PM)<br><input type="radio"/> <b>006</b> M. STACKPOLE ..... (1PM)<br><input type="radio"/> <b>007</b> P. NEWBERRY ..... (3PM) |
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### SHOW ALL YOUR WORK

final answers without any supporting work  
will receive no credit *even if they are right!*

**No calculators allowed.**  
**No cheat-sheets allowed.**

**Partial credit** will be given for any **reasonable amount of work pointing in the right direction** towards the solution of your problem. You will not get any partial credit for memorizing formulas and not knowing how to use them, or for anything you write that is not directly related to the solution of your problem.

If your tests contains **more than one solution or answer** to a problem or part of a problem, and one of them is wrong, then it will be **the wrong one** the one that **counts** for your grading!

**DO NOT WRITE INSIDE THIS BOX!**

problem	points	score
<b>1</b>	15 pts	
<b>2</b>	15 pts	
<b>3</b>	15 pts	
<b>4</b>	15 pts	
<b>5</b>	15 pts	
<b>6</b>	10 pts	
<b>7</b>	15 pts	
<b>TOTAL</b>	100 pts	

1. [15 pts] Let  $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$  be the position vector of the point  $(x, y, z)$ . Show that, if  $a$  is a constant,

$$\text{grad}(\|\vec{r}\|^a) = a\|\vec{r}\|^{a-2}\vec{r}, \quad \vec{r} \neq \vec{0}.$$

2. [15 pts] Approximate  $\pi$  using

$$\pi = f(C, r) = \frac{C}{2r},$$

where the circumference of the circle  $C$  and the radius  $r$  are given by

$$C = 60 \pm 6 \text{ cm}, \quad \text{and} \quad r = 10 \pm 1 \text{ cm}.$$

Estimate the maximum error in your approximation.

3. [15 pts] Given  $z = f(x, y)$ ,  $x = x(u, v)$ ,  $y = y(u, v)$  and  $x(1, 2) = 5$ ,  $y(1, 2) = 3$ , calculate  $z_u$  at  $(u, v) = (1, 2)$  in terms of some of the numbers  $p, m, c, t, a, k, h, q$ , where

$$\begin{aligned} f_x(1, 2) &= p, & f_y(1, 2) &= c, & x_u(1, 2) &= a, & y_u(1, 2) &= h, \\ f_x(5, 3) &= m, & f_y(5, 3) &= t, & x_v(1, 2) &= k, & y_v(1, 2) &= q. \end{aligned}$$

4. [15 pts] The equation  $F(x, y, z) = k$  defines  $z$  implicitly as a function of  $x$  and  $y$ . Find  $z_x$  and  $z_y$  in terms of  $F_x$ ,  $F_y$  and  $F_z$ .

5. [15 pts] The surface  $S$  is represented by the equation  $w = 0$ , where  $w = F(x, y, z) = x^2 - \frac{y}{z^2}$ . Find an equation for the tangent plane to  $S$  at the point  $(1, 1, 1)$ .

6. [10 pts] Compute the critical points of  $f(x, y) = 2x^2 - 3xy + 8y^2 + x - y$  and classify them.

7. [15 pts] Use the method of *Lagrange multipliers* to find the maximum and minimum values of  $f(x, y) = x^3 - y^2$  subject to the constraint  $g(x, y) = x^2 + y^2 = 1$ .