Final Practice

MATH 2400-004, CALCULUS III, FALL 2019

Name:

1. Let $\vec{\mathbf{F}}(x, y, z) = \langle x, y, xe^z \rangle$ and let S be the part of the cylinder $x^2+y^2 = 1$ beneath the plane $z = 2$ and in the first octant, with positive orientation. Calculate $\displaystyle\iint_S \vec{\mathbf{F}} \cdot d\vec{\mathbf{S}}$.

2. Let $\vec{\mathbf{F}}(x, y, z) = \langle e^{\sin(y)}, xz + 2, \sin(y)x^2 + z \rangle$ and let S be the paraboloid $y = 1 - x^2 - z^2$ with $y \geq 0$, with positive orientation. Calculate $\iint_S \vec{\mathbf{F}} \cdot d\vec{\mathbf{S}}$.

3. Let $\vec{\mathbf{F}}(x, y, z) = \langle ye^{xy} - zy, xe^{xy} - xz, -xy \rangle$ and let C be the intersection of the cylinder $z^2 + y^2 = 9$ and the paraboloid $x = y^2 + z^2$, oriented clockwise. Calculate $\displaystyle\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$.

4. Let $\vec{\mathbf{F}}(x, y, z) = \langle ye^{xy} - yz, xe^{xy} - xz, 1 - xy\rangle$, and let C be the curve $\vec{r}(t) = \langle t\cos(t), t\sin(t), t\rangle$ with $0 \leq t \leq 2\pi$. Calculate $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$.

5. Let $\vec{\mathbf{F}}(x, y, z) = \langle x\cos(x), xy - z, e^z + y\rangle$ and let C be the lines connecting $(1, 0, 0), (1, 1, 0), (1, 1, 1)$, and $(1, 0, 1)$, oriented counterclockwise. Calculate $\displaystyle\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$.

6. Let $\vec{\mathbf{F}}(x, y, z) = \langle y + \cos(x), \cos(y)e^y, z^2 - x\rangle$ and let S be the surface of the region bounded by the parametric equations

$$\vec{\mathbf{r}}(u, v) = \langle \cos(u)(2 + \sin(v)), \sin(u)(2 + \sin(v)), \cos(v)\rangle$$

with $0 \le u \le \frac{\pi}{2}$ and $0 \le v \le 2\pi$ and the plane $y = 0$, positively oriented. Calculate $\displaystyle\iint_S \text{curl } \vec{\mathbf{F}} \cdot d\vec{\mathbf{S}}$.

7. Evaluate $\iint_S \vec{\mathbf{F}} \cdot d\vec{\mathbf{S}}$ where $\vec{\mathbf{F}}(x, y, z) = \langle e^{y^2} + xy, y^2 + z, e^{x^2} - zy \rangle$ and where S is the positively-oriented boundary of the region bounded by and including the following sides: $z = 0$, $x = 1$, $y \geq 0$, $x = y^2$, and $z - x = 1$.

8. Evaluate $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ where $\vec{\mathbf{F}}(x, y, z) = \langle \sin(x^2), yz - e^y, \sin(y^2) - x \rangle$ and C is the boundary (oriented counterclockwise) of the portion of the surface $x^2 + y^2 = 9$ (including $z = 0$) cut off at the plane $y + z = 4$.

9. Evaluate $\iint_S e^z \, dS$ where S is the positively-oriented surface given by $\vec{r}(u,v) = \left\langle \cos(u), \sin(u), \frac{uv}{2} \right\rangle$ with $0 \le u, v \le \pi$.

10. Evaluate $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ where $\vec{\mathbf{F}}(x, y) = \langle \cos^2(x) - y^2x, x^2y + x - e^{y^2} \rangle$ and C is the path starting at $(0, 4)$, going down the y-axis to $(0, -4)$, traveling counterclockwise along $x^2 + y^2 = 16$ to $(4, 0)$, then following $y + x = 4$ back to $(0, 4)$.

11. Evaluate $\displaystyle\iint_S \vec{\mathbf{F}}\cdot d\vec{\mathbf{S}}$ where $\vec{\mathbf{F}}(x,y,z) = \langle -z, e^{x^2}, y^2\rangle$ and S is the positively-oriented surface formed by gluing together the cylinder $x^2+y^2=9$ with $0\le z\le 9$ with the elliptic paraboloid $5+x^2+y^2=z$ with $5\le z\le 9$ and the surface given by the parametrization $\vec{r}(u,v)=\langle u\cos(v), u\sin(v), 9\rangle$ where $0\le v\le 2\pi$ and $2\le u\le 3$.

12. Evaluate $\iint_S \vec{\mathbf{F}} \cdot d\vec{\mathbf{S}}$ where $\vec{\mathbf{F}}(x, y, z) = \langle xe^{y^2}, y\sin(z), 2x - ze^{y^2} \rangle$ S is the surface in the first octant formed by the planes $y = 0$, $x = 0$, and $x + y + z = 1$ oriented outwards.

13. Evaluate $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ where $\vec{\mathbf{F}}(x, y, z) = \langle ze^x, \sin(z), e^x + y\cos(z)\rangle$ and C is the curve given by $\vec{\mathbf{r}}(t) = \langle t^2 \cos(t), t, t\rangle$ where $0 \leq t \leq 2\pi$.

14. Evaluate $\displaystyle\iint_S \operatorname{curl}(\vec{\mathbf{F}}) \cdot d\vec{\mathbf{S}}$ where $\vec{\mathbf{F}}(x,y,z) = \langle y, -x, z\rangle$ and S is the surface, oriented outwards, made of the half-cone $z = \sqrt{x^2+y^2}$ with $0 \le z \le 1$ and the cylinder $x^2+y^2 = 1$ where $1 \le z \le 4$.

15. Evaluate $\displaystyle\int_C x\,ds$ where C is the curve $y = x^2$ in the xy-plane from $0 \leq x \leq 1$.

16. Evaluate $\iint_S \text{curl}(\vec{\mathbf{F}})\cdot d\vec{\mathbf{S}}$ where $\vec{\mathbf{F}} = \langle \cos(e^{y^2+z^2+x^2}), e^{\sin(xy)}, \sin(\cos(\sin(\cos(\sin(xyz)))))\rangle$ and S is the surface of the sphere $x^2 + y^2 + z^2 = 4$ above and including the plane $z = \sqrt{3}$ oriented outward.