Determine if the following statements are TRUE or FALSE and explain.

1. If $z = f(x, y)$ has a local minimum at (a, b) and f is differentiable at (a, b) then $\nabla f(a, b) = \vec{\mathbf{0}}$.
   Solution: True
2. If $z = f(x, y)$ has a local minimum at (a, b), then $\nabla f(a, b) = \vec{\mathbf{0}}$.

   Solution: False. The function can fail to be differentiable, like at the point of a cone
3. If $g(x, y) = 5 - y^2$, then a level curve of g is a parabola.
   Solution: False. The level curves are straight lines
4. If $f(x, y)$ is a scalar function of two variables and $f_x(10, -5)$ is defined, then $f_x(10, -5)$ is a scalar.
   Solution: True
5. There is a function $f(x, y)$ with $\nabla f = \langle y, x\rangle$.
   Solution: True. The function $f(x, y) = xy$ has this property
6. If $z = f(x, y)$ and $z = g(x, y)$ have the same tangent plane at (a, b), then $f = g$.
   Solution: False. If $f(x, y) = x^2 + y^2$ and $g(x, y) = -x^2 - y^2$, they share a tangent plane at $(0, 0)$, but $f \neq g$.
7. If f and g are differentiable functions, then $\nabla(fg) = \nabla f \cdot \nabla g$.
   Solution: False.
8. If the limit as (x, y) approaches $(0, 0)$ of $f(x, y)$ exists and equals 2 for every line going to the origin, then $\lim_{(x,y)\to(0,0)} f(x, y) = 2$.
   Solution: False. It is not sufficient to know that the limit exists and has the same value along *every line*. One must know that the limit exists and has the same value along *every path* through $(0, 0)$
9. Let $f(x, y)$ be a continuous function such that $\frac{\partial}{\partial x} f(x, y) > 0$ for all (x, y) in $\mathbb{R}^2$. Then
   $$\int_{-1}^{0} \int_{0}^{1-x^2} f(x, y)\, dy\, dx = \int_{0}^{1} \int_{0}^{1-x^2} f(x, y)\, dy\, dx$$
   Solution: False. The statement would be true if f is symmetric with respect to the y-axis.
10. A function $f(x, y)$ attains an absolute maximum and absolute minimum on a closed and bounded set.
    Solution: False. The statement would be true if f was continuous.
11. If $D_{\vec{u}} f(a, b) < 0$ for all unit vectors $\vec{u}$, then $f(a, b)$ is a local maximum.
    Solution: True.
12. If $\lim_{(x,y)\to(0,0)} f(x, 0) = 2$ and $\lim_{(x,y)\to(0,0)} f(x, x^2) = -2$, then there must exist a constant K such that
    $$g(x, y) = \begin{cases} f(x, y) & (x, y) \neq (0, 0) \\ K & (x, y) = (0, 0) \end{cases}$$
    and $g(x, y)$ is continuous everywhere.
    Solution: False. Approaching on two different paths gives two different limit values, so the limit $\lim_{(x,y)\to(0,0)} f(x, y)$ does not exist. If the limit does not exist at $(0, 0)$, then f is certainly not continuous at $(0, 0)$.