

Determine if the following statements are TRUE or FALSE and explain.

1. If $z = f(x, y)$ has a local minimum at (a, b) and f is differentiable at (a, b) then $\nabla f(a, b) = \vec{0}$.
2. If $z = f(x, y)$ has a local minimum at (a, b) , then $\nabla f(a, b) = \vec{0}$.
3. If $g(x, y) = 5 - y^2$, then a level curve of g is a parabola.
4. If $f(x, y)$ is a scalar function of two variables and $f_x(10, -5)$ is defined, then $f_x(10, -5)$ is a scalar.
5. There is a function $f(x, y)$ with $\nabla f = \langle y, x \rangle$.
6. If $z = f(x, y)$ and $z = g(x, y)$ have the same tangent plane at (a, b) , then $f = g$.
7. If f and g are differentiable functions, then $\nabla(fg) = \nabla f \cdot \nabla g$.
8. If the limit as (x, y) approaches $(0, 0)$ of $f(x, y)$ exists and equals 2 for every line going to the origin, then $\lim_{(x,y) \rightarrow (0,0)} f(x, y) = 2$.
9. Let $f(x, y)$ be a continuous function such that $\frac{\partial}{\partial x} f(x, y) > 0$ for all (x, y) in \mathbb{R}^2 . Then

$$\int_{-1}^0 \int_0^{1-x^2} f(x, y) dy dx = \int_0^1 \int_0^{1-x^2} f(x, y) dy dx$$

10. A function $f(x, y)$ attains an absolute maximum and absolute minimum on a closed and bounded set.
11. If $D_{\vec{u}}f(a, b) < 0$ for all unit vectors \vec{u} , then $f(a, b)$ is a local maximum.
12. If $\lim_{(x,y) \rightarrow (0,0)} f(x, 0) = 2$ and $\lim_{(x,y) \rightarrow (0,0)} f(x, x^2) = -2$, then there must exist a constant K such that

$$g(x, y) = \begin{cases} f(x, y) & (x, y) \neq (0, 0) \\ K & (x, y) = (0, 0) \end{cases}$$

and $g(x, y)$ is continuous everywhere.