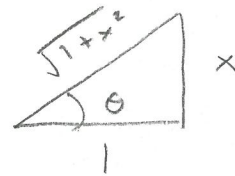


Math 2300 Calculus 2, Spring 2018  
Quiz 3

1.

$$\int \frac{1}{x^2 \sqrt{1+x^2}} dx$$



$$x = \tan \theta$$

$$\frac{dx}{d\theta} = \sec^2 \theta$$

$$\int \frac{1}{\tan^2 \theta \sqrt{1+\tan^2 \theta}} \sec^2 \theta d\theta$$

$$= \int \frac{\sec^2 \theta}{\tan^2 \theta \sec \theta} d\theta = \int \frac{\sec \theta}{\tan^2 \theta} d\theta = \int \frac{1}{\cos \theta} \cdot \frac{\cos^2 \theta}{\sin^2 \theta} d\theta$$

$$= \int \frac{\cos \theta}{\sin^2 \theta} d\theta = -\frac{1}{\sin \theta} + C$$

$$= -\frac{1}{\frac{x}{\sqrt{1+x^2}}} + C = -\frac{\sqrt{1+x^2}}{x} + C$$

2. True/False: If  $\lim_{x \rightarrow \infty} f(x) = 0$ , then  $\int_1^{\infty} f(x) dx$  converges.

(F) If  $h(x) = \frac{1}{\sqrt{x}}$ ,  $\lim_{x \rightarrow \infty} \frac{1}{\sqrt{x}} = 0$

but  $\int_1^{\infty} \frac{1}{\sqrt{x}} dx$  diverges

3. Determine if the following integral converges or diverges:

$$\int_4^{\infty} e^{-x/2} dx.$$

$$\int_4^{\infty} e^{-x/2} dx = \lim_{b \rightarrow \infty} \int_4^b e^{-x/2} dx = \lim_{b \rightarrow \infty} \left( -2e^{-x/2} \Big|_4^b \right)$$

$$= \lim_{b \rightarrow \infty} \left( -2e^{-b/2} - -2e^{-4/2} \right) = 2e^{-2}$$

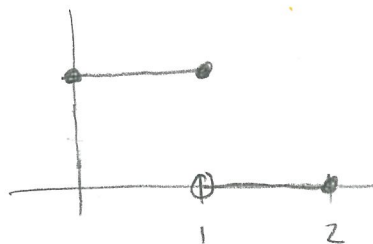
$\downarrow$   
 0

converges

4. Define a piecewise function  $f(x)$  on the interval  $[0, 2]$  by

$$f(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{if } 1 < x \leq 2. \end{cases}$$

Calculate  $L_2$ ,  $R_2$ ,  $M_2$ , and  $T_2$ .



$$L_2 = 1 \cdot 1 + 1 \cdot 1 = 2$$

$$R_2 = 1 \cdot 1 + 0 \cdot 1 = 1$$

$$T_2 = \frac{L_2 + R_2}{2} = \frac{3}{2}$$

$$M_2 = 1 \cdot 1 + 0 \cdot 1 = 1$$