

CALCULUS REVIEW/PREVIEW UNIT 6 SOLUTIONS

$$\begin{aligned}
 1. \quad \int \frac{1}{x^2+a^2} dx &= \int \frac{1}{a^2} \frac{1}{\frac{x^2}{a^2}+1} dx \\
 &= \frac{1}{a^2} \int \frac{1}{\left(\frac{x}{a}\right)^2+1} dx \quad \begin{cases} u = \frac{x}{a} \\ du = \frac{1}{a} dx \\ a du = dx \end{cases} \\
 &= \frac{1}{a^2} \int \frac{a}{u^2+1} du \\
 &= \frac{1}{a^2} \cdot a \cdot \arctan u + c = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + c
 \end{aligned}$$

$$\begin{aligned}
 2. \quad \int \frac{1}{\sqrt{a^2-x^2}} dx &= \int \frac{1}{\sqrt{a^2\left(1-\frac{x^2}{a^2}\right)}} dx = \frac{1}{a} \int \frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}} dx \quad \begin{cases} u = \frac{x}{a} \\ du = \frac{1}{a} dx \\ a du = dx \end{cases} \\
 &= \frac{1}{a} \int \frac{a}{\sqrt{1-u^2}} du = \arcsin(u) + c = \arcsin\left(\frac{x}{a}\right) + c
 \end{aligned}$$

$$\begin{aligned}
 3. \quad \int \frac{1}{x \sqrt{a^2\left(\frac{x^2}{a^2}-1\right)}} dx & \\
 &= \frac{1}{a} \int \frac{1}{x \sqrt{\left(\frac{x}{a}\right)^2-1}} dx \quad \text{To do the } u/du \text{ substitution, I need to} \\
 & \quad \text{make the } x \text{ look like } \frac{x}{a} \text{ somehow...} \\
 &= \frac{1}{a} \int \frac{1}{a \cdot \frac{x}{a} \sqrt{\left(\frac{x}{a}\right)^2-1}} dx \\
 &= \frac{1}{a^2} \int \frac{1}{\frac{x}{a} \sqrt{\left(\frac{x}{a}\right)^2-1}} dx \quad \begin{cases} u = \frac{x}{a} \\ du = \frac{1}{a} dx \\ a du = dx \end{cases} \\
 &= \frac{1}{a^2} \int \frac{a}{u \sqrt{u^2-1}} du \\
 &= \frac{a}{a^2} \operatorname{arcsec}(u) + c = \frac{a}{a^2} \operatorname{arcsec}\left(\frac{x}{a}\right) + c = \frac{1}{a} \operatorname{arcsec}\left(\frac{x}{a}\right) + c
 \end{aligned}$$

$$4. \quad \int \frac{1}{\sqrt{12-x^2}} dx = \arcsin\left(\frac{x}{\sqrt{12}}\right) + c$$

$$5. \quad \int \frac{1}{x \sqrt{x^2-36}} dx = \int \frac{1}{x \sqrt{x^2-6^2}} dx = \frac{1}{6} \operatorname{arcsec}\left(\frac{x}{6}\right) + c$$

$$6. \quad \int \frac{3}{x^2+6} dx = 3 \int \frac{dx}{x^2+(\sqrt{6})^2} = \frac{3}{\sqrt{6}} \arctan\left(\frac{x}{\sqrt{6}}\right) + c$$

$$7. \quad \frac{1}{7} \int \frac{2}{x^2+\frac{1}{7}} dx = \frac{2}{7} \int \frac{dx}{x^2+\left(\frac{1}{\sqrt{7}}\right)^2} = \frac{2\sqrt{7}}{7} \arctan\left(\frac{x}{\sqrt{7}}\right) + c$$