

Here are some commonly used Taylor series. You should know these by heart or be able to compute them quickly.

Function	Taylor series (at $x = 0$)	Interval of convergence
$\frac{1}{1-x}$	$\sum_{n=0}^{\infty} x^n$	$(-1, 1)$
e^x	$\sum_{n=0}^{\infty} \frac{x^n}{n!}$	$(-\infty, \infty)$
$\sin x$	$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$	$(-\infty, \infty)$
$\cos x$	$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$	$(-\infty, \infty)$
$\ln(1+x)$	$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n$	$(-1, 1]$
$\tan^{-1}(x)$	$\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}$	$[-1, 1]$
$(1+x)^p$	$\sum_{n=0}^{\infty} \binom{p}{n} x^n$	$(-1, 1)$

Recall that for any real number p and positive integer n , we define

$$\binom{p}{0} = 1 \quad \text{and} \quad \binom{p}{n} = \frac{p(p-1)(p-2)(p-3)\cdots(p-(n-1))}{n!}.$$

For example,

$$\binom{-4.3}{6} = \frac{(-4.3)(-5.3)(-6.3)(-7.3)(-8.3)(-9.3)}{6!} \approx 112.3663514.$$

When p is also a positive integer, this reduces to our formula for the binomial coefficient,

$$\binom{p}{n} = \frac{p!}{n!(p-n)!}.$$

For example,

$$\binom{7}{4} = \frac{7!}{4!(7-4)!} = \frac{7!}{4! \cdot 3!} = \frac{\cancel{1} \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{4} \cdot 5 \cdot 6 \cdot 7}{\cancel{1} \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{4} \cdot 1 \cdot 2 \cdot 3} = \frac{5 \cdot \cancel{6} \cdot 7}{\cancel{6}} = 5 \cdot 7 = 35.$$