Warm-Up:

1. Evaluate the following integrals

(a) $\int x^2(\sqrt{x}+5)+e^2\,dx$

Solution:

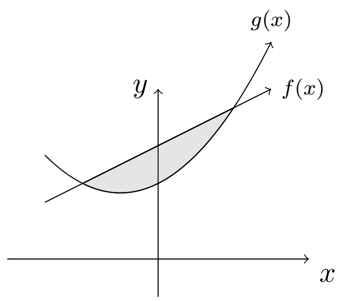
$$\int x^2(\sqrt{x}+5)+e^2\,dx=\int x^{5/2}+5x^2+e^2\,dx=\frac{2}{7}x^{7/2}+\frac{5}{3}x^3+xe^2+c.$$

(b) $\int_1^2 \frac{3x^3+1}{4x}\,dx$

Solution:

$$\int_1^2 \frac{3x^3+1}{4x}\,dx=\int_1^2 \frac{3}{4}x^2+\frac{1}{4x}\,dx=\left[\frac{x^3}{4}+\frac{1}{4}\ln|x|\right]_1^2=\frac{8}{4}+\frac{1}{4}\ln 2-\frac{1}{4}-\frac{1}{4}\ln 1=\frac{7}{4}+\frac{1}{4}\ln 2.$$

2. Find the area bounded between the line $f(x)=x+3$ and the parabola $g(x)=x^2+x+2$.



(a) Find where $f(x)$ and $g(x)$ intersect by setting them equal and solving for x.

Solution: We want to find x so that $x+3=x^2+x+2$. This amounts to finding where

$$x^2-1=0.$$

From factoring, we see that this is true when $x=\pm1$. So $f(x)$ and $g(x)$ intersect at $x=\pm1$.

(b) Set up the integral and evaluate to find the area bounded by $f(x)$ and $g(x)$.

Solution: From the graph we can see that $f\geq g$ on $[-1,1]$. So the area between the curves is

$$\int_{-1}^1 f(x)-g(x)\,dx=\int_{-1}^1 x+3-(x^2+x+2)\,dx=\int_{-1}^1 -x^2+1\,dx=\left[-\frac{x^3}{3}+x\right]_{-1}^1=-\frac{1}{3}+1-\frac{1}{3}+1=\frac{4}{3}.$$

3. Evaluate the following indefinite integrals using substitution.

   (a) $\displaystyle\int \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx$

   Solution: Let $u = \sqrt{x}$. Then $du = \frac{1}{2\sqrt{x}}dx$. Substituting, we have

$$\int \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx = \int \cos(\sqrt{x}) \cdot \frac{dx}{\sqrt{x}} = \int \cos(u) \cdot 2\,du = 2\sin(u) + c = 2\sin(\sqrt{x}) + c.$$

   (b) $\displaystyle\int \frac{e^x}{e^x+1}\,dx$

   Solution: Let $u = e^x + 1$. Then $du = e^x dx$. Substituting, we have

$$\int \frac{e^x}{e^x+1}\,dx = \int \frac{1}{e^x+1} \cdot e^x\,dx = \int \frac{1}{u}\,du = \ln|u| + c = \ln|e^x+1| + c.$$

4. Evaluate the following definite integrals using substitution.

   (a) $\displaystyle\int_2^3 \frac{xe^{x^2}}{3}\,dx$

   Solution: Let $u = x^2$. Then $du = 2xdx$, or $\frac{du}{2} = xdx$. Substituting, we have

$$\int_2^3 \frac{xe^{x^2}}{3}\,dx = \frac{1}{3}\int_2^3 e^{x^2} \cdot xdx = \frac{1}{3}\int_{x=2}^3 e^u \cdot \frac{du}{2} = \frac{1}{6}\int_{x=2}^3 e^u\,du = \frac{1}{6}\left[e^u\right]_{x=2}^3 = \frac{1}{6}(e^9 - e^4).$$

   (b) $\displaystyle\int_0^1 \frac{x}{1+3x^2}dx$

   Solution: Let $u = 1 + 3x^2$. Then $du = 6xdx$, or $\frac{du}{6} = xdx$. Substituting, we have

$$\int_0^1 \frac{x}{1+3x^2}\,dx = \int_0^1 \frac{1}{1+3x^2} \cdot xdx = \int_{x=0}^1 \frac{1}{u} \cdot \frac{du}{6} = \frac{1}{6}\int_{x=0}^1 \frac{du}{u} = \frac{1}{6}\ln|u|\Big|_{x=0}^1 = \frac{1}{6}\ln|1+3x^2|\Big|_0^1 = \frac{\ln 4}{6}.$$

5. Show the following two integrals are equivalent:

$$\int_0^2 3x\sqrt{9-x^2}dx = \int_5^9 \frac{3\sqrt{u}}{2}du.$$

Solution: Let's do a u-substitution on the left. Let $u = 9 - x^2$. Then $du = -2xdx$, or $-\frac{du}{2} = xdx$. The bounds are $x = 0$ and $x = 2$, which transform into $u = 9$ and $u = 5$. Substituting, we have

$$\int_0^2 3x\sqrt{9-x^2}\,dx = 3\int_0^2 \sqrt{9-x^2} \cdot xdx = 3\int_9^5 \sqrt{u} \cdot -\frac{du}{2} = -\frac{3}{2}\int_9^5 \sqrt{u}\,du = \frac{3}{2}\int_5^9 \sqrt{u}\,du,$$

which is equal to the right-hand side.