Taylor Series Matching – Solutions

$\boxed{\mathrm{A}, 6, \mathrm{j}}$
$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$\boxed{\mathrm{B}, 7, \mathrm{e}}$
$$e^{2x} = \sum_{n=0}^{\infty} \frac{2^n x^n}{n!} = 1 + 2x + \frac{4x^2}{2} + \frac{8x^3}{3!} + \frac{16x^4}{4!} + \cdots$$

$\boxed{\mathrm{C}, 11, \mathrm{a}}$
$$e^{-x^2} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!} = 1 - x^2 + \frac{x^4}{2} - \frac{x^6}{3!} + \frac{x^8}{4!} - \cdots$$

$\boxed{\mathrm{D}, 10, \mathrm{d}}$
$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots$$

$\boxed{\mathrm{E}, 4, \mathrm{f}}$
$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots$$

$\boxed{\mathrm{F}, 9, \mathrm{i}}$
$$x \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n)!} = x - \frac{x^3}{2} + \frac{x^5}{4!} - \frac{x^7}{6!} + \frac{x^9}{8!} - \cdots$$

$\boxed{\mathrm{G}, 2, \mathrm{h}}$
$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \cdots$$

$\boxed{\mathrm{H}, 3, \mathrm{m}}$
$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n = 1 - x + x^2 - x^3 + x^4 - \cdots$$

$\boxed{\mathrm{J}, 5, \mathrm{k}}$
$$\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n} = 1 - x^2 + x^4 - x^6 + x^8 - \cdots$$

$\boxed{\mathrm{K}, 12, \mathrm{c}}$
$$\frac{d}{dx}\left(\frac{1}{1-x}\right) = \frac{1}{(1-x)^2} = \sum_{n=1}^{\infty} n x^{n-1} = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + \cdots$$

$\boxed{\mathrm{L}, 8, \mathrm{b}}$
$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots$$

$\boxed{\mathrm{M}, 1, \mathrm{g}}$
$$\int_0^x \frac{1}{1+t^2}\, dt = \arctan x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \cdots$$