

1. In Yellowstone park there were 21 bison in 1902, and 250 in 1915. Using the model that the rate of change of the population is proportional to the population itself, set up an initial value problem to model this situation. You don't need to solve.

**Solution:** Letting  $P(t)$  be the population, we have  $\frac{dP}{dt} = kP$ . Letting  $t = 0$  in 1902, we have that  $P(0)=21$  and  $P(13)=250$ .

2. The world record times in the women's mile run can be predicted with the model that the times drop at a rate proportional to the difference between 4 minutes and the current record time. Anne Rosemary Smith set the record with a time of 4:37 in June of 1967, and Svetlana Masterkova holds the current record (set in August 1996) of 4:12.56. Set up an initial value problem to model this situation. You don't need to solve.

**Solution:** Letting  $R(t)$  represent the current world record time, we have that  $\frac{dR}{dt} = k(R-4)$ . Differentiating the left-hand side, we have  $\frac{dR}{dt} = k(R-4)$ . Letting  $t = 0$  in June of 1967, we have  $R(0) = 4 + \frac{37}{60}$  and  $R(29 + 2/12) = 4 + \frac{12.56}{60}$ .

3. Data shows that if we define  $P(t)$  to be the total amount of oil produced in the US since 1859 (the year the first oil well was built), then a logistic model is a reasonable model for  $P(t)$ . The limiting value is predicted to be 180 billion barrels. The amount produced by 1950 was 40.9 billion barrels, and the amount produced by 1969 was about 90 billion barrels. Set up (but don't solve) the initial value problem.

**Solution:** The logistic model is  $\frac{dP}{dt} = kP(1 - \frac{P}{L}) = kP(1 - \frac{P}{180})$ . Letting  $t = 0$  in 1950, we have  $P(0) = 40.9$  and  $P(19) = 90$ .

4. A savings account earns 6% annual interest, compounded continuously (so the growth rate of the account due to interest is .06 times the balance). Money is deposited continuously at a rate of \$1500 per year. The initial balance is \$0. Write a differential equation for  $B(t)$ , the balance in the account.

**Solution:**  $\frac{dB}{dt} = .06B + 1500$ ,  $B(0) = 0$ .