Developing your intuition: For each of the following series, guess if it diverges, converges conditionally or converges absolutely. Keep in mind that you must answer two separate questions: 1. Does the series converge? and 2. Does the series converge absolutely? Name the test(s) you would use to answer each of these questions. Usually you are required to give a detailed solution, but for this worksheet, just briefly describe your overall strategy.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(n+\frac{1}{2})}{n-\frac{1}{2}}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\, n}{e^n}$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n!}$

4. $\displaystyle\sum_{n=1}^{\infty} \frac{(\sin n)2^n}{n!}$

5. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n(n^3+1)}{n^4+n-4}$

6. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^{3/2}}$

7. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3+n}$

8. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n \arctan n}{\sqrt{n}}$

9. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n^2}$

10. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n\, n}{(\ln n)^2}$

11. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n^7+n}}{\sqrt{n^9+n^5}}$

12. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n^7+n}}{\sqrt{n^{10}+n^5}}$

13. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\, 10n^2}{n^4+1}$

14. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

15. $\displaystyle\sum_{n=1}^{\infty} \frac{n(-2)^n}{n!}$

16. $\displaystyle\sum_{n=1}^{\infty} \frac{2-5^n}{11^{n-1}(-1)^n}$

17. $\displaystyle\sum_{n=1}^{\infty} \sqrt{n}2^{n+1}$

18. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{4n^5+n^4-1}}$

19. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3^n\, n!}{1\cdot 3\cdot 5\cdot 7 ... (2n-1)}$

20. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \sin(n^3)}{2^n}$

21. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\, n!}{e^{n^2}}$

Answers:

1. Diverges.
(divergence test)

2. Converges absolutely.
(ratio test or integral test)

3. Converges absolutely.
(ratio test)

4. Converges absolutely.
First show $\sum \frac{2^n}{n!}$ converges using the ratio test, then compare the absolute value of our series to $\sum \frac{2^n}{n!}$ using term-size comparison.

5. Converges conditionally.
Use A.S.T to show convergence. Then take the absolute value and use L.C.T. (compare to $\sum b_n = \sum \frac{1}{n}$) to show convergence is NOT absolute.

6. Converges absolutely.
Compare to p-series $\sum \frac{1}{n^{3/2}}$ using term-size comparison.

7. Converges absolutely.
Take absolute value, use L.C.T., and compare to $\sum \frac{1}{n^3}$

8. Converges conditionally.
Use A.S.T to show convergence and L.C.T with $\sum \frac{1}{\sqrt{n}}$ to show convergence is not absolute.

9. Converges absolutely.
Take absolute value, then either: compare term-wise to $\sum \frac{\sqrt{n}}{n^2}$
or: use the integral test (integrate by parts with $u = \ln n$).

10. Diverges.
(divergence test)

11. Converges conditionally.
Use A.S.T to show convergence and then take absolute value and compare to $\sum \frac{1}{n}$ to show that convergence is not absolute (L.C.T.).

12. Converges absolutely.
Take absolute value, then compare to $\sum \frac{1}{n^{3/2}}$ using limit comparison

13. Converges absolutely.
Either compare to $\sum \frac{1}{n^2}$ using limit comparison, or compare to $\sum \frac{10}{n^2}$ using term-size comparison.

14. Diverges.
Use integral test (integrate by substition with $u = \ln n$).

15. Converges absolutely.
(ratio test)

16. Converges absolutely.
(Break the difference into two separate series, each is a geometric series, $|r| < 1$)

17. Diverges.
(divergence test)

18. Converges absolutely.
Compare to $\sum \frac{1}{n^{5/2}}$.

19. Diverges.
(ratio test, careful with the cancellations)

20. Converges absolutely.
Take absolute value and then compare to $\sum \frac{1}{2^n}$.

21. Converges absolutely.
(ratio test)