

Turn in the following problems at the start of your Thursday recitation section. To receive full credit, please staple your work, and put your name, your section number, and the homework number at the top.

1. (a) Use the integral test to show that $\sum_{n=1}^{\infty} 1/n^4$ converges.
- (b) Find the 10th partial sum, s_{10} , of the series $\sum_{n=1}^{\infty} 1/n^4$.
- (c) According to the Remainder Estimate for the Integral Test, we know that

$$\int_{n+1}^{\infty} \frac{1}{x^4} dx \leq s - s_n \leq \int_n^{\infty} \frac{1}{x^4} dx, \quad (1)$$

where s is the sum of $\sum_{n=1}^{\infty} 1/n^4$ and s_n is the n th partial sum of $\sum_{n=1}^{\infty} 1/n^4$. Use inequality (1) and s_{10} from part (b) to find an upper and lower bound for s .

- (d) Use the Remainder Estimate for the Integral Test to find a value of n so that s_n is within 0.00001 of the sum.
2. Find the sum of the series $\sum_{n=1}^{\infty} 1/n^5$ correct to three decimal places. Be sure to justify your answer.
3. Show that the series $\sum_{k=1}^{\infty} (-1)^{k-1} k e^{-k}$ is convergent. How many terms of the series do we need to add in order to find the sum to within 0.01 of its true value (i.e. so that $|\text{error}| < 0.01$).

(4-9) Determine whether each series is absolutely convergent, conditionally convergent or divergent. Carefully show your work.

$$4. \sum_{n=1}^{\infty} \frac{n!}{99^n}$$

$$5. \sum_{k=1}^{\infty} k \left(\frac{5}{6}\right)^k$$

$$6. \sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2 + 1}$$

$$7. \sum_{k=1}^{\infty} \frac{(-3)^k k!}{(2k)!}$$

$$8. \sum_{k=1}^{\infty} \frac{\sin 4k}{2^k}$$

$$9. \sum_{n=1}^{\infty} (-1)^n \frac{2^n}{\sqrt{n}}$$