

Turn in the following problems at the start of your Thursday recitation section. To receive full credit, please staple your work, and put your name, your section number, and the homework number at the top.

(1-4) Use the Comparison Test or Limit Comparison Test to determine whether each series is convergent or divergent. Carefully explain your reasoning.

1.
$$\sum_{k=2}^{\infty} \frac{2k^3}{k^4 - 6}$$

2.
$$\sum_{k=0}^{\infty} \frac{1 + \cos k}{8^k}$$

3.
$$\sum_{k=1}^{\infty} \frac{4k^2 - k}{k^3 + k + 1}$$

4.
$$\sum_{k=1}^{\infty} \frac{1}{k^2 \arctan(k)}$$

5. Show that if $a_k > 0$ for $k \geq 1$ and $\sum_{k=1}^{\infty} a_k$ converges, then $\sum_{k=1}^{\infty} \ln(1 + a_k)$ also converges.

6. Suppose $\frac{1}{k^2} \leq a_k \leq \frac{1}{k}$ for $k \geq 1$. Is it true that $\sum_{k=1}^{\infty} a_k$ converges? Why or why not?

(7-10) Test each of the following series for convergence or divergence. Carefully explain your reasoning.

7.
$$-\frac{3}{4} + \frac{5}{5} - \frac{7}{6} + \frac{9}{7} - \frac{11}{8} + \dots$$

8.
$$\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{\ln(k+5)}$$

9.
$$\sum_{k=3}^{\infty} (-1)^k \frac{k}{\sqrt{k^3 + 1}}$$

10.
$$\sum_{k=1}^{\infty} (-1)^k \cos\left(\frac{\pi}{k}\right)$$

11. Show that the series

$$\sum_{k=1}^{\infty} \frac{(-1)^k}{k^5 5^k}$$

is convergent. How many terms of the series do we need to add in order to estimate the sum to within 0.0001?