Turn in the following problems at the start of your Thursday recitation section. To receive full credit, please staple your work, and put your name, your section number, and the homework number at the top.

(1-4) Determine whether each series below is convergent or divergent. If the series is convergent, **find its sum**. Carefully show your work, and justify any tests that you use.

1. $\displaystyle\sum_{k=1}^{\infty} \frac{(k+1)(3k-1)}{(k+3)^2}$

2. $\displaystyle\sum_{k=1}^{\infty} \cos(5)^k$

3. $\displaystyle\sum_{k=1}^{\infty} \ln\left(\frac{k}{k+1}\right)$

4. $\displaystyle\sum_{k=3}^{\infty} \frac{1}{k^2+k}$

5. Find the values of x for which the series

$$\sum_{k=0}^{\infty} \frac{(x+2)^k}{7^k}$$

converges. Find the sum of the series for those values of x.

6. A certain ball has the property that each time it falls from a height h onto a hard, level surface, it rebounds to a height rh, where $0 < r < 1$. Suppose that the ball is dropped from an initial height of H meters.

   (a) Assuming that the ball continues to bounce indefinitely, find the total distance that it travels.

   (b) Calculate the total time that the ball travels. (Use the fact that the ball falls $\frac{1}{2}gt^2$ meters in t seconds.)

7. Suppose f is a continuous, positive, decreasing function for $x \geq 1$ and $a_k = f(k)$ for $k \geq 1$. By drawing a picture, rank the following three quantities in increasing order:

$$\int_1^6 f(x)\,dx, \qquad \sum_{k=1}^{5} a_k, \qquad \sum_{k=2}^{6} a_k.$$

8. It is important to distinguish between

$$\sum_{k=1}^{\infty} k^b \qquad \text{and} \qquad \sum_{k=1}^{\infty} b^k.$$

   (a) What is the difference between these two series (what name do we give to each one)?

   (b) For what values of b does the first series converge?

   (c) For what values of b does the second series converge?

(9-14) Determine whether each series below is convergent or divergent. Justify any tests that you use.

9. $\displaystyle\sum_{k=1}^{\infty}(k^{-1.3}+3k^{-1.7})$

10. $1+\dfrac{1}{2\sqrt{2}}+\dfrac{1}{3\sqrt{3}}+\dfrac{1}{4\sqrt{4}}+\dfrac{1}{5\sqrt{5}}+\cdots$

11. $\displaystyle\sum_{k=1}^{\infty}\frac{4+3^k}{2^k}$

12. $\displaystyle\sum_{k=1}^{\infty}\frac{1}{k^2+16}$

13. $\displaystyle\sum_{k=1}^{\infty}ke^{-2k}$

14. $\displaystyle\sum_{k=2}^{\infty}\frac{k}{\ln k}$

15. The following statements are ***false***. For each statement please provide a counterexample:

   (a) If $\lim\limits_{n\to\infty} a_n = 0$, then $\displaystyle\sum_{n}^{\infty} a_n$ converges.

   (b) Let f be a continuous function and define a sequence $a_n = f(n)$. If $\lim\limits_{n\to\infty} a_n$ exists and is equal to a finite real number L, then $\lim\limits_{x\to\infty} f(x) = L$.