

Turn in the following problems at the start of your Thursday recitation section. To receive full credit, please staple your work, and put your name, your section number, and the homework number at the top.

1. A certain small country has \$10 billion in paper currency in circulation. Each day \$50 million of the money in circulation enters the country's banks, and another \$50 million leaves the banks and enters circulation. The government decides to introduce new currency by having the banks replace the old bills with the new ones whenever old currency comes into the banks. Let $x = x(t)$ denote the amount of new currency in circulation at time t , with $x(0) = 0$. Assume that the proportion of new money entering the banks each day is the same as the proportion of new money in circulation. How long would you estimate it to take for the new bills to account for 90% of the currency in circulation?

2. Suppose the position (x, y) of a particle at time t is given by the parametric equations

$$\begin{cases} x(t) = 2 \sin t, \\ y(t) = 4 + \cos t, \end{cases} \quad 0 \leq t \leq 3\pi/2.$$

- (a) Eliminate the parameter t to find a Cartesian equation for the path traced by the particle.
- (b) Draw a graph to depict the motion of the particle for $0 \leq t \leq 3\pi/2$. On your graph, mark the start and end points and the direction of motion of the particle.
3. Suppose the position (x, y) of a particle at time t is given by the parametric equations

$$\begin{cases} x(t) = \sin t, \\ y(t) = \cos^2 t, \end{cases} \quad -2\pi \leq t \leq 2\pi.$$

- (a) Eliminate the parameter t to find a Cartesian equation for the path traced by the particle.
- (b) Draw a graph to depict the motion of the particle for $-2\pi \leq t \leq 2\pi$. On your graph, mark the start and end points and the direction of motion of the particle.
4. (a) Why does the curve with parametric equations $x = \sin t$, $y = \sin(t + \sin t)$ have two tangent lines at the origin?
- (b) Find the equations of both tangent lines.
- (c) Use technology to help you sketch the graph of the curve and its two tangent lines at the origin.
5. Match the parametric equations in Exercises 19–24 from section 10.1 of the course text (page 700).
6. Find a parameterization for the line segment going from the point $(2, 3, -4)$ to the point $(5, -2, -4)$.

7. For the parametric curve $x = t^3 + t$, $y = t^2$, find the equation of the tangent line at $t = 1$, find the speed at $t = 1$, and then find $\frac{d^2y}{dx^2}$ at $t = 1$ to determine if the curve is concave up or concave down there.
8. Set up an integral for the length of the curve $y = \sin x$ between $x = 0$ and $x = \pi$. Use technology to find the length correct to four decimal places.
9. Set up an integral for the length of the curve $y = \ln x$ between $x = 1$ and $x = \sqrt{3}$. Use technology to find the exact length. (For a challenge and a review of trigonometric substitution and partial fractions, try to do this integral by hand!)
10. (a) Use technology to aid you in sketching the graph of $y = \frac{x^3}{3} + \frac{1}{4x}$, for $1 \leq x \leq 2$.
(b) Write down and evaluate (by hand) an integral for the exact length of this curve.