Turn in the following problems at the start of your Thursday recitation section. To receive full credit, please staple your work, and put your name, your section number, and the homework number at the top.

1. Find a closed-form solution to the integral equation

$$y(x) = 3 + \int_e^x \frac{dt}{ty(t)}, \quad x > 0.$$

   In other words, express $y(x)$ as a function that doesn't involve an integral. (Hint: Use the Fundamental Theorem of Calculus to obtain a differential equation. You can find an initial condition by evaluating the original integral equation at a strategic value of x.)

2. A sample of tritium-3 decayed to 94.5% of its original amount after a year.

   (a) What is the half-life of tritium-3?

   (b) How long would it take the sample to decay to 30% of its original amount?

3. A murder occurred on a sunny, 20°C autumn afternoon. The temperature of the corpse was measured to be 32.5°C at 1:30 PM and 30.3°C an hour later. At what time did the murder take place? (Use the fact that the normal temperature of the human body is 37.0°C.)

4. (a) If \$2000 is borrowed at 7% interest, find the amounts due at the end of 3 years if the interest is compounded

       i. annually,

       ii. quarterly,

       iii. monthly,

       iv. weekly,

       v. daily,

       vi. hourly, and

       vii. continuously.

   (b) Based on your answers to part (a), what does it mean to have "continuously" compounded interest?

   (c) If $A(t)$ is the amount of the investment at time t for the case of continuous compounding, write a differential equation and an initial condition satisfied by $A(t)$.

5. (Removed) ~~A certain small country has $10 billion in paper currency in circulation. Each day $50 million of the money in circulation enters the country's banks, and another $50 million leaves the banks and enters circulation. The government decides to introduce new currency by having the banks replace the old bills with the new ones whenever old currency comes into the banks. Let $x = x(t)$ denote the amount of new currency in circulation at time t, with $x(0) = 0$. Assume that the proportion of new money entering the banks each day is the same as the proportion of new money in circulation. How long would you estimate it to take for the new bills to account for 90% of the currency in circulation?~~

6. Biologists stocked a lake with 400 fish and estimated the carrying capacity (the maximal population for the fish of that species in that lake) to be 8000. The number of fish tripled in the first year.

   (a) Assuming that the size of the fish population satisfies the logistic equation, find an expression for the size of the population after t years.

   (b) How long will it take for the population to increase to 4000?