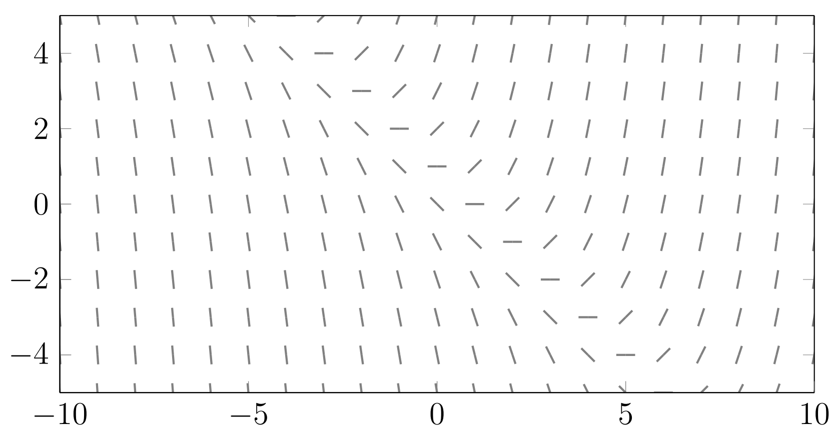
Turn in the following problems at the start of your Thursday recitation section. To receive full credit, please staple your work, and put your name, your section number, and the homework number at the top.

1. (a) For what values of k does the function $y = \cos(kt)$ satisfy the differential equation
$$4y'' = -25y? \tag{1}$$
   (b) For those values of k that you found in part (a), verify that every function of the form
$$y = A\sin(kt) + B\cos(kt)$$
   is also a solution to the differential equation (1). (Here, A and B are constants.)

2. A function $y(t)$ satisfies the differential equation
$$\frac{dy}{dt} = y^4 - 6y^3 + 5y^2.$$
   (a) What are the constant solutions of the equation? (Recall that these have the form $y = C$ for some constant, C.)
   (b) For what values of y is y increasing?
   (c) For what values of y is y decreasing?

3. For the differential equation $y' = x + y - 1$, whose direction field is given below, sketch the graphs of the solutions that satisfy the initial conditions

   (a) $y(0) = -1$, (b) $y(0) = 0$, (c) $y(0) = 1$.



4. (a) Sketch a direction field for the differential equation
$$y' = x - y + 1,$$
   and use it to sketch three solution curves of your choice.
   (b) For each of the three solution curves that you sketched in part (a), write down an initial condition that gives rise to that solution curve.

5. In this problem, we will investigate the differential equation $y' = xy - x^2$ graphically using a direction field and numerically via Euler's method.

   (a) Sketch a direction field for the differential equation

   $$y' = xy - x^2$$

   and use it to sketch a solution curve that passes through $(0, 1)$.

   (b) Use Euler's method with step size 0.2 to estimate $y(1)$, where $y(x)$ is the solution of the initial-value problem

   $$y' = xy - x^2, \; y(0) = 1.$$

6. Solve the differential equations

   (a) $\dfrac{du}{dr} = \dfrac{1+\sqrt{r}}{1+\sqrt{u}}$, (b) $(y^2 + xy^2)y' = 1.$

   (Note: you do not need to solve for u as a function of r in your solution to part (a). Recall from Calculus I that in this situation, we say u is an *implicit* function of r. You could use implicit differentiation to check your work.)

7. Find the solution of the initial value problem

   $$y' = \frac{xy \sin x}{y+1}, \; y(0) = 1.$$

   (Note: you do not need to solve for y as a function of x in your final answer.)

8. Find the function f such that $f'(x) = f(x)(1 - f(x))$ and $f(0) = 1/2$.

9. Review related rates before Project 12, and get a head start on the "warm-up" sections of this project. (See Section 4.1 of the course text; see also the Related Rates Review posted under the Calculus 1 review resources section of the More Resources tab of the course webpage.)