Turn in the following problems at the start of your Thursday recitation section. To receive full credit, please staple your work, and put your name, your section number, and the homework number at the top.

1. Do, but don't turn in: memorize the nth degree Taylor polynomials centered at $a = 0$ for e^x, $\sin(x), \cos(x), \ln(1 + x)$, and $\frac{1}{1-x}$. Be able to write them down with ease in both expanded form and sigma notation.

2. This problem asks for Taylor polynomials for $f(x) = \ln(1 + x)$ centered at $a = 0$. Show your work in an organized way.

   (a) Find the 4th, 5th, and 6th degree Taylor polynomials for $f(x) = \ln(1 + x)$ centered at $a = 0$.

   (b) Find the nth degree Taylor polynomial for $f(x)$ centered at $a = 0$, **written in expanded form**.

   (c) Find the nth degree Taylor polynomial for $f(x)$ centered at $a = 0$, **written in sigma (summation) notation**.

   (d) Use the 7th degree Taylor polynomial to estimate $\ln(2)$.

   (e) Compare your answer to the estimate for $\ln(2)$ given by your calculator. How accurate were you?

   (f) Looking at the Taylor polynomials, explain why this estimate is less accurate than the estimate you found for $\sin(3°)$ in Problem 6 of Homework 10. (In that problem, you were asked to use a 7th degree Taylor polynomial centered at $a = 0$ for $\sin(x)$ to approximate $\sin(3°)$).

3. We wish to estimate $\ln(0.5)$ using an nth degree Taylor polynomial for $\ln(1 + x)$ centered at $a = 0$. How large should n be to guarantee the approximation will be within 0.0001? (Hint: Start by calculating a formula for $|f^{(n+1)}(z)|$ and finding a bound on this quantity between $x = -1/2$ and $a = 0$.)

   (If you need some help, click here for a sample Taylor's inequality problem, and click here for the solutions. These can be found by visiting the "More Resources" tab of the course webpage and looking under the "Resources for Calculus 2" heading.)

4. Use the Maclaurin series for $\sin(x)$ to compute $\sin(3°)$ to within 0.000005. Be sure to include a careful explanation of how you chose n, the number of terms to add.

5. Prove that the Taylor series for $f(x) = \sin(x)$ centered at $a = \pi/2$ represents $\sin(x)$ for all x. In other words, show that $\lim_{n\to\infty} R_n(x) = 0$ for each x, where $R_n(x)$ is the remainder between $\sin(x)$ and the nth degree Taylor polynomial for $\sin(x)$ centered at $a = \pi/2$.

6. Find the Maclaurin series for each of the functions below. [Hint: In each case, start with one of the common Maclaurin series that you memorized in Problem 1 above. See Table 1 in Section 8.7 of the course text.]

   (a) $f(x) = 3e^x + e^{-x}$ (b) $g(x) = x^5 \ln(1 + x^2)$

7. Use an appropriate Taylor series to approximate the definite integral $\int_0^{0.5} x^2 e^{-x^2}\,dx$ to within 0.001 of the true value.

8. Find the sum of the series
$$1 - \ln 5 + \frac{(\ln 5)^2}{2!} - \frac{(\ln 5)^3}{3!} + \cdots .$$

9. Find the Taylor series for xe^x about $x = 0$. Then, integrate term-by-term and substitute to show that the series $\sum_{n=0}^{\infty} \frac{1}{n!(n+2)}$ converges to 1.

10. (a) Find $T_4(x)$, the 4th degree Taylor polynomial for $f(x) = \sin x$ centered at $a = \pi/6$.

    (b) Use Taylor's inequality to estimate the accuracy of the approximation $f(x) \approx T_4(x)$ when $0 \le x \le \pi/3$. In other words, what is the biggest error we could have if we were to use $T_4(x)$ in place of $f(x) = \sin x$ on the interval $[0, \pi/3]$?

11. Use Taylor's inequality to estimate the range of values of x for which the approximation
$$\cos x \approx 1 - \frac{x^2}{2} + \frac{x^4}{24}$$
is accurate to within 0.005.