Turn in the following problems at the start of your Thursday recitation section. To receive full credit, please staple your work, and put your name, your section number, and the homework number at the top.

1. Do, but don't turn in: Memorize the formula for the nth-degree Taylor Polynomial for $f(x)$ centered at a:

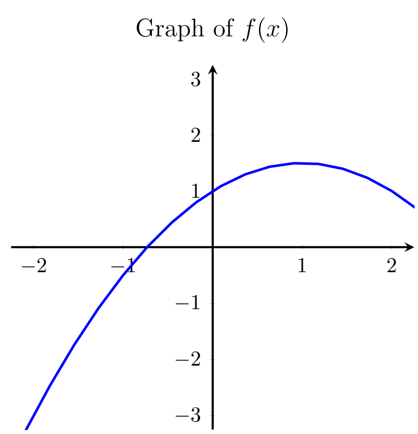
$$\begin{aligned} T_n(x) &= f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots + \frac{f^{(n)}(x)}{n!}(x-a)^n \\ &= \sum_{i=0}^{n} \frac{f^{(i)}(a)}{i!}(x-a)^i \end{aligned}$$

2. Find the 4th degree Taylor polynomial for $\tan(x)$ centered at $a = 0$.

3. Suppose that a function $f(x)$ is approximated near $a = 0$ by the 3rd degree Taylor polynomial $T_3(x) = 4 - 3x + \frac{x^2}{5} + 4x^3$. Give the values of $f(0), f'(0), f''(0)$, and $f'''(0)$.

4. Find the 10th degree Taylor polynomial centered at $a = 1$ for $f(x) = 2x^2 - x + 1$.

5. Suppose a function $f(x)$ has the following graph:



If the 2nd degree Taylor polynomial centered at $a = 0$ for $f(x)$ is $T_2(x) = bx^2 + cx + d$, determine the signs (i.e. $+/-$) of b, c and d.

6. The goal of this problem is to use a Taylor polynomial to approximate $\sin(3°)$. Show your work in an organized manner.

   (a) Find the 7th degree Taylor polynomial centered at $a = 0$ for $\sin(x)$.

   (b) Use $T_7(x)$ to estimate $\sin(3°)$. Don't forget to convert to radians.

   (c) Compare your estimate for $\sin(3°)$ to the value that technology gives you. How accurate is the approximation you found?

(7-9) Find the radius of convergence and interval of convergence of each series.

7. $\displaystyle\sum_{n=1}^{\infty} \frac{(3n)!}{3^n} x^n$
8. $\displaystyle\sum_{n=1}^{\infty} \frac{(3x-1)^n}{n2^n}$
9. $\displaystyle\sum_{n=2}^{\infty} \frac{x^{2n}}{n(\ln n)^2}$

10. Suppose that $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = -5$ and diverges when $x = 7$. What can be said about the convergence or divergence of the following series?
    (a) $\displaystyle\sum_{n=0}^{\infty} c_n$
    (b) $\displaystyle\sum_{n=0}^{\infty} c_n(-2)^n$
    (c) $\displaystyle\sum_{n=0}^{\infty} (-1)^n c_n 9^n$
    (d) $\displaystyle\sum_{n=0}^{\infty} c_n 6^n$

11. Find a power series representation for the function $f(x) = \frac{x}{2x^2+1}$ and determine the interval of convergence.

12. (a) Use the power series for $\frac{1}{1-x}$ to find a power series representation of $f(x) = \ln(1-x)$. What is the radius of convergence? (Note: you don't need to use the ratio test here because we know the radius of convergence of the series $\sum_{n=0}^{\infty} x^n$.)
    (b) Use part (a) to find a power series for $f(x) = x\ln(1-x)$.
    (c) By putting $x = \frac{1}{2}$ in your result from part (a), express $\ln 2$ as the sum of an infinite series.

13. Use a power series to approximate the definite integral $\displaystyle\int_0^{0.3} \frac{x^2}{1+x^4}\,dx$ to six decimal places.