

Final Exam Review

1. $\int e^x \cos x \, dx$
2. $\int x \ln x \, dx$
3. $\int \cos^3 x \, dx$
4. $\int \sec^3 x \, dx$
5. $\int \sin^3 x \cos^2 x \, dx$
6. $\int_2^3 \frac{1}{x^2 - 1} \, dx$
7. $\int \frac{10}{(x-1)(x^2+9)} \, dx$
8. $\int \frac{1}{x^2 \sqrt{1-x^2}} \, dx$
9. $\int \sqrt{1+x^2} \, dx$
10. $\int_0^1 \ln x \, dx$
11. $\int_0^\infty x e^{-x} \, dx$
12. Set up, but do not evaluate an integral which computes the volume of the solid generated by revolving the region bounded by $y = \cos x$, $y = 2 - \cos x$, $0 \leq x \leq 2\pi$ about the line $y = 4$.
13. Compute the arc length of the curve $\begin{cases} x = e^t \cos t \\ y = e^t \sin t \end{cases}$ for $0 \leq t \leq \pi$.
14. Find the average value of the function $f(x) = 2 \sin x - \sin(2x)$ over the interval $[0, \pi]$.
15. Find the amount of work necessary to drain a full spherical tank with radius 3m by pumping water to the top of the tank.
16. Solve $y' = \frac{xy \sin x}{y+1}$, $y(0) = 1$.
17. Solve $y' \cot^2 x = 1 + y$, $y(\pi/3) = 1$ when $0 \leq x \leq \pi/2$.
18. A vat with 500 gallons of beer contains 4% alcohol by volume. Beer with 6% alcohol is pumped into the vat at a rate of 5 gallons per minute and a well-mixed mixture leaves the tank at the same rate. Find the alcohol content (by volume) of the beer after 1 hour.

19. A roasted turkey is taken out of an oven when its average temperature is 180 F. It is placed on a table in a room where the temperature is 75 F. If the temperature is 150 F after half-an-hour, what will the temperature be after 45 minutes?
20. Suppose a population develops according to the logistic equation

$$\frac{dP}{dt} = 0.05P - 0.0005P^2.$$

- (a) What is the carrying capacity of the environment?
- (b) If $P(0) = 10$, what is $P'(0)$?
21. Find the interval of convergence for $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n \cdot 3^n}$.
22. Express $f(x) = \frac{x}{2x^2+1}$ as a power series centered at 0.
23. Converges or Diverges? $\sum_{n=0}^{\infty} \frac{1}{(\sqrt{2})^n}$
24. Converges or Diverges? $\sum_{n=0}^{\infty} \arctan(n)$
25. Converges or Diverges? $\sum_{n=0}^{\infty} \frac{n^3+1}{3n^4-1}$
26. Converges or Diverges? $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$
27. Converges or Diverges? $\sum_{n=0}^{\infty} \frac{2^n \cdot n^3}{n!}$
28. Converges or Diverges? $\sum_{n=0}^{\infty} \frac{\sqrt{n}}{1+n^2}$

29. Find the sum of the convergent series $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{4^n (2n)!}$

30. Find the sum of the convergent series $\sum_{n=1}^{\infty} \frac{3^n}{n \cdot 5^n}$

31. Find the sum of the convergent series $\sum_{n=1}^{\infty} \frac{3^n}{5^n \cdot n!}$

32. Find the slope of the line tangent to the curve at the corresponding point.

$$\begin{cases} x = \cos \theta + \sin 2\theta \\ y = \sin \theta + \cos 2\theta \end{cases}, \theta = \pi/3$$

33. For what values of t , $0 < t < \pi$, is the curve $\begin{cases} x = \cos 2t \\ y = \cos t \end{cases}$ concave upward?

34. Find a Cartesian curve equivalent to $r = 3 \sin \theta$.

35. Find a Cartesian curve equivalent to $r = \tan \theta \sec \theta$.

36. Find the slope of the tangent to $r = \cos 2\theta$ when $\theta = \pi/4$.

37. Find the area enclosed by $r = 1 + \cos \theta$.

38. Find the area inside the outer loop, but outside the inner loop to $r = \frac{1}{2} + \cos \theta$.

39. Find the arc length of $r = 3 \sin \theta$, $0 \leq \theta \leq \pi/3$.

40. Find the arc length of $r = \theta^2$, $0 \leq \theta \leq 2\pi$.