FINAL EXAM
CALCULUS 2

MATH 2300
FALL 2018

Name _____

PRACTICE EXAM

Please answer all of the questions, and show your work.
You must explain your answers to get credit.
You will be graded on the clarity of your exposition!

Date: December 12, 2018.

1
10 points

1. Consider the region bounded by the graphs of $f(x) = x^2 + 1$ and $g(x) = 3 - x^2$.

1.(a). *(5 points)* Write the integral for the volume of the solid of revolution obtained by rotating this region about the x-axis. Do not evaluate the integral.

1.(b). *(5 points)* Write the integral for the volume of the solid of revolution obtained by rotating this region about the line $x = 3$. Do not evaluate the integral.

2
6 points

2. MULTIPLE CHOICE: Circle the best answer.

2.(a). *(1 point)* Is the integral $\int_{-1}^{1} \frac{1}{x^2}\, dx$ an improper integral?

Yes No

2.(b). *(5 points)* Evaluate the integral: $\int_{-1}^{1} \frac{1}{x^2}\, dx =$

3
14 points

3. Consider the curve parameterized by $\begin{cases} x = \frac{1}{3}t^3 + 3t^2 + \frac{2}{3} \\ y = t^3 - t^2 \end{cases}$ for $0 \leq t \leq \sqrt{5}$.

3.(a). *(6 points)* Find an equation for the line tangent to the curve when $t = 1$.

3.(b). *(3 points)* Compute $\frac{d^2y}{dx^2}$ at $t = 1$.

3.(c). *(5 points)* Write an integral to compute the total arc length of the curve. Do not evaluate the integral.

4
8 points

4. Consider the function $f(x) = x^2 \arctan(x)$.

4.(a). *(5 points)* Find a power series representation for $f(x)$.

4.(b). *(3 points)* What is $f^{(83)}(0)$, the 83rd derivative of $f(x)$ at $x = 0$?

5. A tank contains 200 L of salt water with a concentration of 4 g/L. Salt water with a concentration of 3 g/L is being pumped into the tank at the rate of 8 L/min, and the tank is being emptied at the rate of 8 L/min. Assume the contents of the tank are being mixed thoroughly and continuously. Let $S(t)$ be the amount of salt (measured in grams) in the tank at time t (measured in minutes).

5
10 points

5.(a). *(1 points)* What is the amount of salt in the tank at time $t = 0$?

5.(b). *(2 points)* What is the rate at which salt enters the tank?

5.(c). *(2 points)* What is the rate at which salt leaves the tank at time t?

5.(d). *(1 points)* What is $\frac{dS}{dt}$, the net rate of change of salt in the tank at time t?

5.(e). *(4 points)* Write an initial value problem relating $S(t)$ and $\frac{dS}{dt}$. Solve the initial value problem.

6
8 points

6. Compute the following integrals.

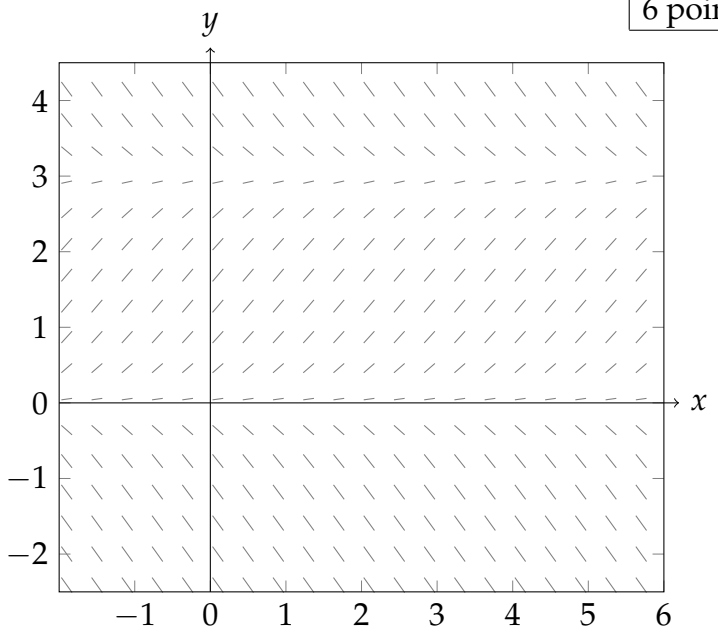
6.(a). *(4 points)* $\int \sin^3(x)\cos^2(x)\ dx$

6.(b). *(4 points)* $\int \dfrac{x+1}{x^2(x-1)}\ dx$

7. A slope field for the differential equation $y' = 2y\left(1 - \frac{y}{3}\right)$ is shown below.

7
6 points

$$y' = 2y\left(1 - \frac{y}{3}\right)$$



7.(a). *(2 points)* Sketch the graph of the solution that satisfies following initial condition. Label the solution as (a).

$$y(0) = 1$$

7.(b). *(2 points)* Sketch the graph of the solution that satisfies following initial condition. Label the solution as (b).

$$y(0) = -1$$

7.(c). *(2 points)* Show that for $y(0) = c \geq 0$, we have $\lim_{x\to\infty} y(x)$ is finite.

8
6 points

8. Consider the series $\sum_{n=1}^{\infty} \frac{1}{n^4}$.

8.(a). *(3 points)* Use the Remainder Estimate for the Integral Test to find an upper bound for the error in using S_{10} (the 10th partial sum) to approximate the sum of this series.

8.(b). *(3 points)* How many terms suffice to ensure that the sum is accurate to within 10^{-6}?

9
12 points

9. Determine whether the series is convergent or divergent and circle the corresponding answer. Then write the test allows one to determine convergence or divergence

9.(a). *(3 points)* $\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}}$

convergent divergent

Test:

9.(b). *(3 points)* $\sum_{n=1}^{\infty} \frac{(-1)^n(n+1)}{n^2-3}$

convergent divergent

Test:

9.(c). *(3 points)* $\sum_{n=1}^{\infty} \cos\left(\frac{5}{n}\right)$

convergent divergent

Test:

9.(d). *(3 points)* $\sum_{n=1}^{\infty} \frac{n^2+5}{(n+2)!}$

convergent divergent

Test:

10
6 points

10. MULTIPLE CHOICE: Circle the best answer below.

10.(a). *(2 points)* The sequence $a_n = 1 - 0.2^n$

converges to 0. converges, but not to 0. diverges.

10.(b). *(2 points)* The sequence $a_n = \dfrac{3n-4}{2n-1}$

converges to 0. converges, but not to 0. diverges.

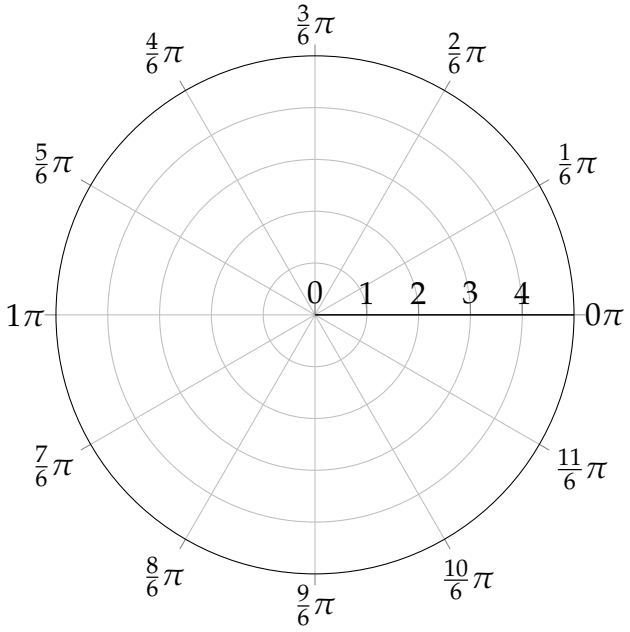
10.(c). *(2 points)* The sequence $a_n = n + \dfrac{1}{n}$

converges to 0. converges, but not to 0. diverges.

11
8 points

11.

11.(a). *(4 points)* Sketch the curves $r = 2$ and $r = 3 + 2\sin\theta$ on the axes below.



11.(b). *(4 points)* Write an integral that represents the area contained outside the first curve ($r = 2$) and inside the second curve ($r = 3 + 2\sin(\theta)$). Do not evaluate the integral.

12
8 points

12. MULTIPLE CHOICE: Circle the best answer below.

12.(a). *(2 points)* Is the following statement ALWAYS, SOMETIMES, or NEVER true?

If $\sum |a_n|$ converges, then $\sum a_n$ converges.

ALWAYS SOMETIMES NEVER

12.(b). *(2 points)* Is the following statement ALWAYS, SOMETIMES, or NEVER true?

If $\sum a_n$ converges, then $\sum |a_n|$ converges.

ALWAYS SOMETIMES NEVER

12.(c). *(2 points)* The graph of $\begin{cases} x = t^2 - 3 \\ y = -t \end{cases}$ for $-\infty < t < \infty$ is a

line parabola circle ellipse

12.(d). *(2 points)* The graph of $\begin{cases} x = t^2 - 3 \\ y = -t^2 \end{cases}$ for $-\infty < t < \infty$ is a

line parabola circle ellipse