Math 2300, Final

December 15, 2015

PRINT YOUR NAME: ______________________________

PRINT INSTRUCTOR'S NAME: ______________________________

Mark your section/instructor:

☐	Section 001	Albert Bronstein	9:00 - 9:50
☐	Section 002	Andrew Healy	10:00 - 10:50
☐	Section 003	Joshua Frinak	11:00 - 11:50
☐	Section 004	Kevin Berg	12:00 - 12:50
☐	Section 005	Jeffrey Shriner	2:00 - 2:50
☐	Section 006	Megan Ly	3:00 - 3:50
☐	Section 007	Albert Bronstein	8:00 - 8:50
☐	Section 008	Jonathan Lamar	1:00 - 1:50
☐	Section 009	Keli Parker	3:00 - 3:50
☐	Section 010	Steven Weinell	4:00 - 4:50
☐	Section 011	Albert Bronstein	8:00 - 8:50

Question	Points	Score
1	6	
2	8	
3	8	
4	8	
5	8	
6	8	
7	8	
8	6	
9	8	
10	8	
11	8	
12	8	
13	8	
Total:	100	

- No calculators or cell phones or other electronic devices allowed at any time.
- Show all your reasoning and work for full credit. Use full mathematical or English sentences.
- You have 150 minutes and the exam is 100 points.
- You do not need to simplify numerical expressions. For example leave fractions like 100/7 or expressions like $\ln(3)/2$ as is.
- When done, give your exam to your instructor, who will mark your name off on a photo roster.
- We hope you show us your best work!

1. (6 points) Circle the correct answer.

   (a) Determine the partial fraction decomposition of $\dfrac{x+3}{(x^2-1)(x+2)}$.

   (i) $\dfrac{A}{x-1}+\dfrac{B}{x^2-1}+\dfrac{C}{x+2}$

   (ii) $\dfrac{A}{x-1}+\dfrac{B}{x+2}$

   (iii) $\dfrac{A}{x^2-1}+\dfrac{B}{x+2}$

   (iv) $\dfrac{A}{x-1}+\dfrac{B}{x+1}+\dfrac{C}{x+2}$

   (b) Which trigonometric substitution is needed to integrate $\displaystyle\int \frac{\sqrt{4-x^2}}{x^2}dx$?

   (i) $x = 2\tan(\theta)$

   (ii) $x = 4\tan(\theta)$

   (iii) $x = 2\sin(\theta)$

   (iv) $x = 4\sin(\theta)$

2. (8 points) Evaluate the following indefinite integral. Show all work.

$$\int \arctan(x)\,dx$$

3. (8 points) Compute the sum of

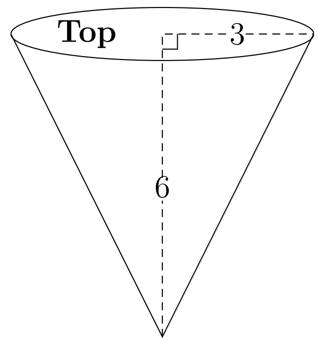
$$\sum_{n=1}^{\infty} \frac{5 \cdot 2^{n+1}}{3^{2n-1}}.$$

You do not need to simplify.

4. (8 points) Setup **but do not compute** an integral to find the volume of the solid of revolution determined by rotating the region bounded by $x = 0$, $y = 2$ and $y = \sqrt{x}$ about the x-axis.

$$\int_{\boxed{}}^{\boxed{}} \boxed{}$$

5. (8 points) Setup **but do not compute** an integral to find the work done pumping the water out of the top of a full tank with the shape of a right circular cone with height 6 meters, radius 3 meters. Express the acceleration due to gravity as g and the density of water as ρ.



$$\int_{\boxed{}}^{\boxed{}} \boxed{}$$

6. (8 points) Determine whether the series converges absolutely, converges conditionally, or diverges. Rigorously justify your answer.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$$

7. (8 points) Consider the function $f(x) = e^{3x^2}$.

(a) Write the Maclaurin series for $f(x)$.

(b) Use the Maclaurin series for $f(x)$ to express $\int_0^1 f(x)\,dx$ as a series.

8. (6 points) Circle the correct answer.

(a) Find the interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{3^n}{\sqrt{n}} x^n$.

(i) $\left[-\frac{1}{3}, \frac{1}{3}\right)$

(ii) $\left(-\frac{1}{3}, \frac{1}{3}\right]$

(iii) $\left(-\frac{1}{3}, \frac{1}{3}\right)$

(iv) $\left[-\frac{1}{3}, \frac{1}{3}\right]$

(v) $(-\infty, \infty)$

(b) Using the power series

$$\ln(x+1) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$$

and the Estimation Theorem for the Alternating Series, we conclude that the least number of terms in the series needed to approximate $\ln 2$ with error $< 3/1000$ is:

(i) 333

(ii) 534

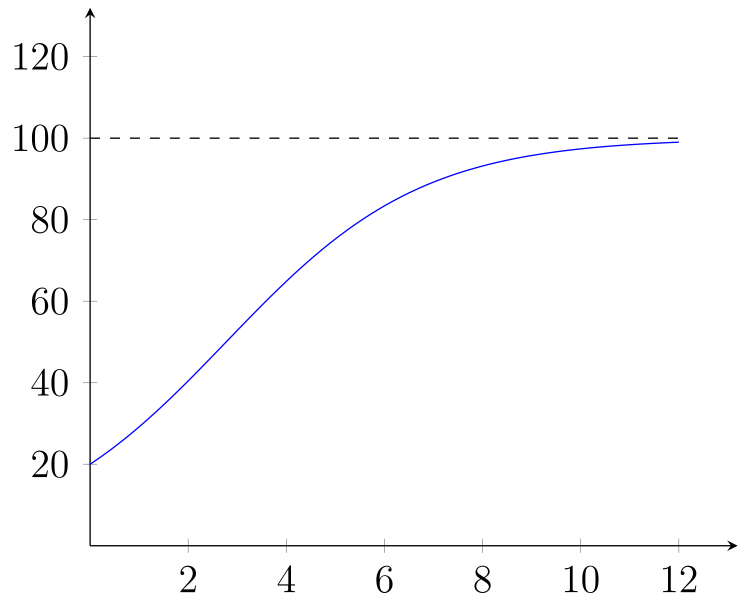
(iii) 100

(iv) 9

(v) 201

9. (8 points) The following graph is a particular solution to the logistic differential equation

$$\frac{dP}{dt} = .5P\left(1 - \frac{P}{100}\right).$$



(a) Determine the carrying capacity, M, and the initial population, P_0.

$M =$

$P_0 =$

(b) Which function below represents the particular solution above?

(i) $P(t) = \dfrac{1}{1+4e^{-.5t}}$

(ii) $P(t) = \dfrac{100}{1+4e^{-.5t}}$

(iii) $P(t) = \dfrac{100}{1+20e^{-.5t}}$

(iv) $P(t) = \dfrac{1}{1+20e^{-.5t}}$

10. (8 points) Consider the parametric curve

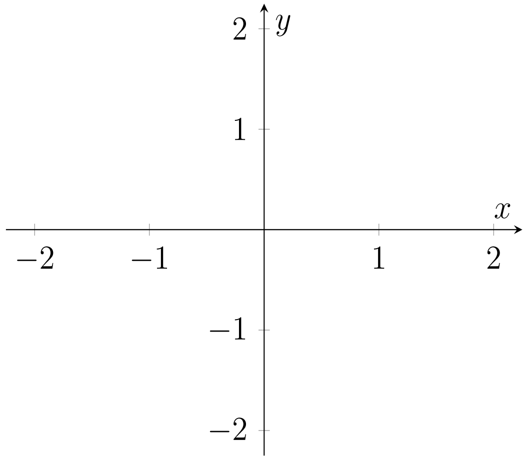
$$\begin{aligned} x(t) &= t^2 \\ y(t) &= 2t + 2. \end{aligned}$$

(a) Find the equation for the tangent line to the parametric curve at $t = 2$.

(b) Is the parametric curve concave up or down at $t = 2$? Justify your answer.

11. (8 points) Follow the given steps to find the area of one petal of the rose curve given by $r = 2\sin(2\theta)$.

(a) Sketch the graph of $r = 2\sin(2\theta)$ on the interval $0 \leq \theta \leq 2\pi$.



(b) Fill in the boxes below to set up, **but not evaluate**, an integral which gives the area of one petal of the rose curve given by $r = 2\sin(2\theta)$.

$$\int_{\boxed{}}^{\boxed{}} \boxed{}$$

12. (8 points) Determine whether the following improper integral converges. If the integral does converge, evaluate it.

$$\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\, dx$$

13. (8 points) Find $\dfrac{dy}{dx}$ of the polar function $r = 2\cos(3\theta)$.