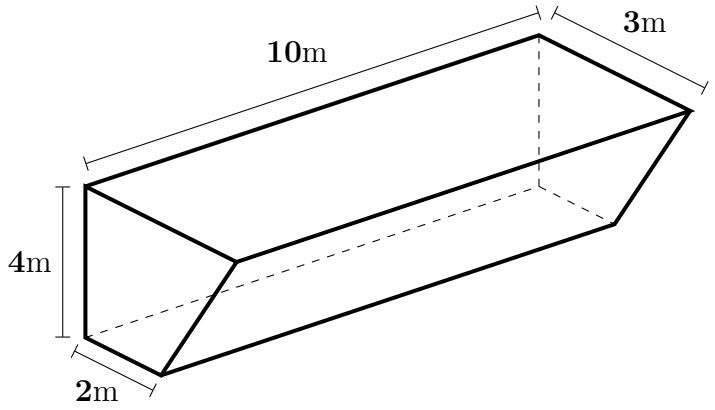
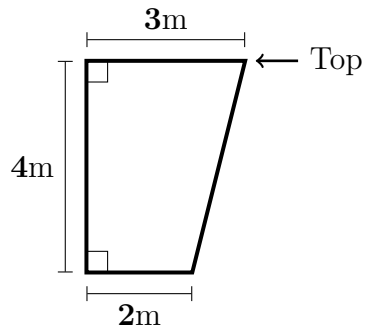
1. Water is pumped over the top of the full tank depicted below. How much work is done to drain the tank? Use $\boldsymbol{\rho}$ kg/m$^{\mathbf{3}}$ for the density of water and $\boldsymbol{g}$ m/sec$^{\mathbf{2}}$ for the acceleration due to gravity.

Side View:



Front View:



(A) $\frac{\mathbf{340}}{\mathbf{3}}\boldsymbol{\rho g}$ **J** (B) $\frac{\mathbf{380}}{\mathbf{3}}\boldsymbol{\rho g}$ **J** (C) $\frac{\mathbf{650}}{\mathbf{3}}\boldsymbol{\rho g}$ **J** (D) $\frac{\mathbf{560}}{\mathbf{3}}\boldsymbol{\rho g}$ **J** (E) $\frac{\mathbf{520}}{\mathbf{3}}\boldsymbol{\rho g}$ **J**

2. Consider a lamina of uniform density bounded by the lines $\boldsymbol{x + y = 1}$, $\boldsymbol{x = 0}$, and $\boldsymbol{y = 0}$. Compute the center of mass $\boldsymbol{(\bar{x}, \bar{y})}$ of the lamina.

(A) $\left(\frac{\mathbf{1}}{\mathbf{3}}, \frac{\mathbf{1}}{\mathbf{3}}\right)$ (B) $\left(\frac{\mathbf{2}}{\mathbf{5}}, \frac{\mathbf{2}}{\mathbf{5}}\right)$ (C) $\left(\frac{\mathbf{3}}{\mathbf{7}}, \frac{\mathbf{3}}{\mathbf{7}}\right)$ (D) $\left(\frac{\mathbf{1}}{\mathbf{4}}, \frac{\mathbf{1}}{\mathbf{4}}\right)$ (E) $\left(\frac{\mathbf{4}}{\mathbf{9}}, \frac{\mathbf{4}}{\mathbf{9}}\right)$

3. Find the average value of the function $\boldsymbol{f(x) = x^3 + x}$ on the interval $\boldsymbol{[0, 2]}$.

(A) **1** (B) **2** (C) **3** (D) **4** (E) **5** (F) **6**

4. A force of **100** N is required to hold a spring that has been stretched from its natural length of **20** cm to a length of **25** cm. How much work is done in stretching the spring from **25** cm to **30** cm?

(A) **15** J (B) **10** J (C) **5** J (D) **5.5** J (E) **7.5** J (F) **9.5** J

5. Find the volume of the solid of revolution obtained by revolving the region bounded by the graphs of $\boldsymbol{y = \sin x,\ 0 \leq x \leq \pi}$, and the $\boldsymbol{x}$-axis, about the $\boldsymbol{y}$-axis.

(A) $\boldsymbol{\pi}$ (B) $\boldsymbol{2\pi}$ (C) $\boldsymbol{\pi^2}$ (D) $\boldsymbol{2\pi^2}$ (E) $\boldsymbol{\pi(\pi - 1)}$ (F) $\boldsymbol{\pi^2(\pi - 1)}$

6. Consider the curve with parametric equations $x = \sin t$, $y = \dfrac{2}{\sin t}$.
   Its tangent has slope -1 when $t =$

   (A) $\dfrac{\pi}{6}$ (B) $\dfrac{\pi}{2}$ (C) $\dfrac{\pi}{4}$ (D) $\dfrac{\pi}{3}$ (E) π (F) There is no such t

7. The length of the curve $x = 2t + \sin t$, $y = 1 - \cos t$, $(0 \le t \le \pi)$ is

   (A) $\displaystyle\int_0^\pi \sqrt{4t^2 + 2 + 4t\sin t - 2\cos t}\, dt$

   (B) $\displaystyle\int_0^\pi \sqrt{2 - 2\cos t + \cos^2 t}\, dt$

   (C) $\displaystyle\int_0^\pi \sqrt{1 + \sin^2 t}\, dt$

   (D) $\displaystyle\int_0^\pi \sqrt{3 + 2\cos t + \sin^2 t}\, dt$

   (E) $\displaystyle\int_0^\pi \sqrt{5 + 4\cos t}\, dt$

8. The length of the curve given by $y = \dfrac{x^{3/2} - x^{1/2}}{\sqrt{3}}$, $1 \le x \le 4$ is

   (A) $2\sqrt{3}$ (B) $\dfrac{8}{\sqrt{3}}$ (C) 6π (D) 8π (E) $\dfrac{\pi}{\sqrt{3}}$

9. A conical tank is 5 meters high and the radius of its base is 2 meters long. The base of the tank rests on the ground. If the tank is filled with a liquid of density ρ kg/m^3 and g m/sec^2 is gravitational acceleration, the work necessary to empty it by pumping the liquid through its vertex at the top is

   (A) $20\pi\rho g$ J (B) $16\pi\rho g$ J (C) $50\pi\rho g$ J (D) $25\pi\rho g$ J (E) $12.5\pi\rho g$ J

10. If a curve has parametric equation given by $x = e^{2t}$, $y = e^{-t}$, then $\dfrac{d^2y}{dx^2}$, for $t = 0$, is equal to

(A) 0 (B) 1 (C) $\dfrac{2}{3}$ (D) $\dfrac{1}{4}$ (E) $\dfrac{1}{3}$

11. If we use the method of cylindrical shells, then the volume of the solid obtained by rotating the region bounded by the curves

$$y = e^{-x^2},\ y = 0,\ x = 3,\ x = 5$$

about the line $x = 2$ is expressed by integration:

(A) $2\pi \displaystyle\int_3^5 (x-2)e^{-x^2}\,dx$

(B) $2\pi \displaystyle\int_3^5 xe^{-x^2}\,dx$

(C) $2\pi \displaystyle\int_0^1 y\sqrt{-\ln y}\,dy$

(D) $2\pi \displaystyle\int_{-5}^5 xe^{-x^2}\,dx$

(E) $2\pi \displaystyle\int_3^5 (x+2)e^{-x^2}\,dx$

12. Which of the following is a solution to the differential equation $y' = \dfrac{y}{x\ln x}$?

(A) $y = \ln x$ (B) $y = x\ln x$ (C) $y = \dfrac{\ln x}{x}$ (D) $y = x^2\ln x$ (E) $y = \dfrac{\ln x}{x^2}$

13. The length of the arc of the spiral given in polar coordinates by $r = e^{-2\theta}$, $0 \le \theta \le 1/2$ is

(A) $\dfrac{e+1}{\sqrt{6}}$ (B) $\dfrac{1-e}{2\sqrt{3}}$ (C) $4(1-e^{-1})$ (D) $\sqrt{2}(e^2-1)$ (E) $\dfrac{\sqrt{5}(e-1)}{2e}$

14. If $y(x)$ is the solution to the initial value problem $y' = \dfrac{4xy}{2+x^2}$, $y(0) = 4$, then $y(1) =$

(A) 2 (B) 4 (C) 9 (D) 1 (E) 0

15. Find the volume of the solid of revolution obtained by rotating the region bounded by the graphs of $y = \ln x, y = 0, x = 1,$ and $x = e$ about the y-axis.

(A) $\dfrac{\pi}{2}(e^2-1)$ (B) $\dfrac{\pi}{2}(e^2+1)$ (C) $\dfrac{\pi}{2}(e^2-3)$ (D) $\pi(e^2+1)$ (E) $\pi(e^2-1)$

16. The area inside the curve $r = 3\sin\theta$ and outside the curve $r = 1+\sin\theta$ is given by

(A) $\dfrac{1}{2}\displaystyle\int_{\pi/3}^{2\pi/3}(8\sin^2\theta - 1 - 2\sin\theta)\,d\theta$

(B) $\dfrac{1}{2}\displaystyle\int_{\pi/3}^{2\pi/3}(4\sin^2\theta - 4\sin\theta + 1)\,d\theta$

(C) $\dfrac{1}{2}\displaystyle\int_{\pi/6}^{5\pi/6}(4\sin^2\theta - 4\sin\theta + 1)\,d\theta$

(D) $\dfrac{1}{2}\displaystyle\int_{\pi/6}^{5\pi/6}(8\sin^2\theta - 1 - 2\sin\theta)\,d\theta$

(E) $\dfrac{1}{2}\displaystyle\int_{\pi/6}^{5\pi/6}(4\sin^2\theta + 4\sin\theta + 1)\,d\theta$

17. Find the slope of the tangent line to $x = te^{-t}$, $y = \dfrac{t^3}{3}$, at $t = 2$.

(A) $\dfrac{1}{e^2}$ (B) $4e^2$ (C) $-4e^2$ (D) $-\dfrac{4}{e^2}$ (E) e^2

18. The length of the curve $r = \sin^3\theta$, $0 \le \theta \le \pi$ is:

(A) $\displaystyle\int_0^\pi \sin\theta\sqrt{\sin\theta + 3\cos\theta}\,d\theta$

(B) $\displaystyle\int_0^\pi \sin\theta\sqrt{\sin\theta + 9\sin^2\theta\cos^2\theta}\,d\theta$

(C) $\displaystyle\int_0^\pi \sin\theta\sqrt{\sin^4\theta + 3\cos\theta}\,d\theta$

(D) $\displaystyle\int_0^\pi \sin^2\theta\sqrt{1 + 8\cos^2\theta}\,d\theta$

(E) $\displaystyle\int_0^\pi \sin^2\theta\sqrt{\sin^2\theta - 9\cos^2\theta}\,d\theta$

19. A hard-boiled egg at $98°\text{C}$ is put in a sink of $18°\text{C}$ water. After 5 minutes, the egg's temperature is $38°\text{C}$. Assuming that the water has not wormed appreciably, how much longer will it take the egg to reach $20°\text{C}$?

(A) 5 min (B) 8 min (C) 11 min (D) 13 min (E) 16 min (F) 23 min

20. Find the area of the region inside both $r = 8\cos\theta$ and $r = 8\sin\theta$.

(A) $8(\pi - 2)$ (B) $8(\pi + 2)$ (C) $\dfrac{8}{3}(4\pi - 3\sqrt{3})$ (D) $4(2 - \sqrt{2})$ (E) 16π

21. Find the length of the curve $r = e^{4\theta}$ with $0 \le \theta \le \pi$.

(A) $\dfrac{1}{16}(e^{8\pi} - 1)$

(B) $\dfrac{\sqrt{17}}{4}(e^{4\pi} - 1)$

(C) $\dfrac{32}{3}(\pi^2 - 1)$

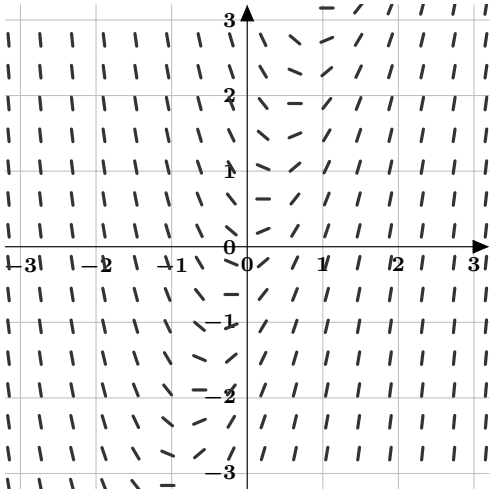
(D) $(e^{8\pi} - \sqrt{3})$

(E) $\sqrt{17}(e^{4\pi} - 1)$

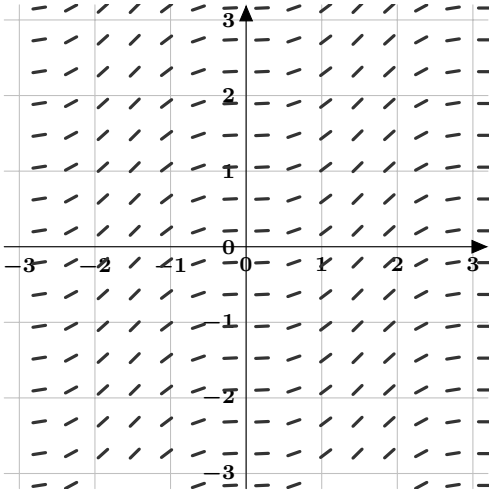
22. Identify which of the following slope fields corresponds to the differential equation

$$\frac{dy}{dx} = x + 2y$$

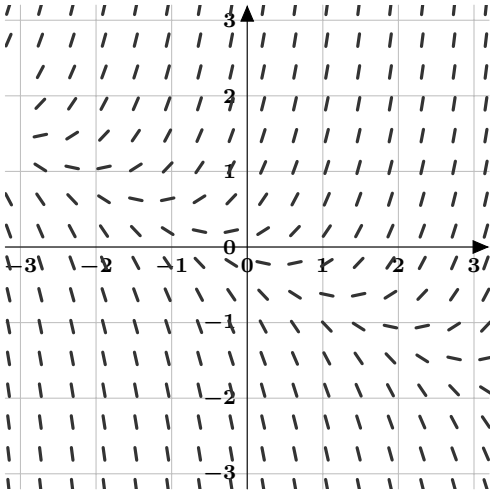
I



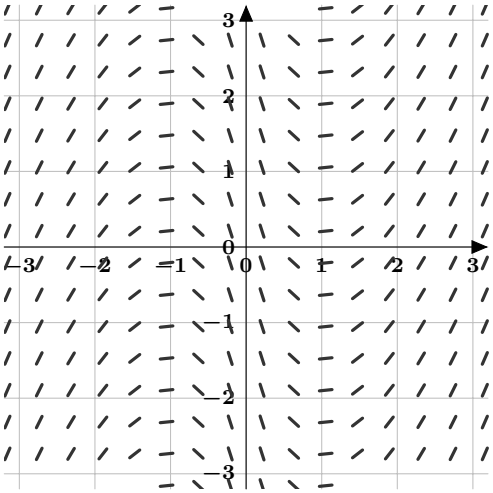
II



III



IV



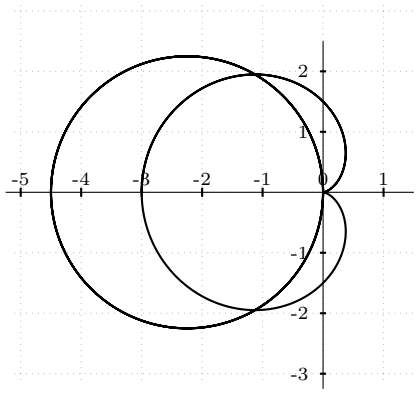
23. A curve is given by parametric equations $x = t + \ln t,\ y = t - \ln t$.

When $t = 1$ then $\dfrac{d^2y}{dx^2} =$

(A) $\dfrac{8}{3}$ (B) $\dfrac{7}{4}$ (C) $\dfrac{2}{5}$ (D) $\dfrac{1}{4}$ (E) 0

24. Select the correct graph of the circle $r = 3\cos\theta$ and the cardiod $r = 1 + \cos\theta$.

I)



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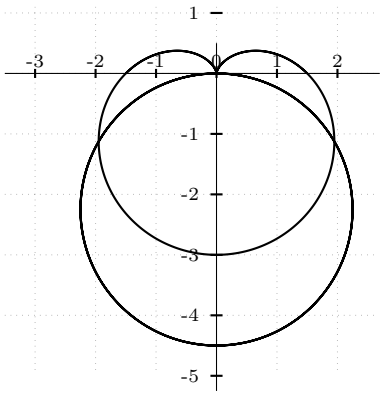
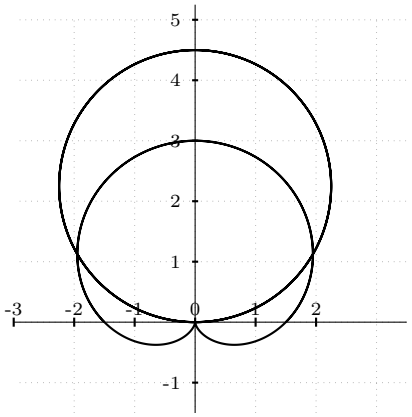
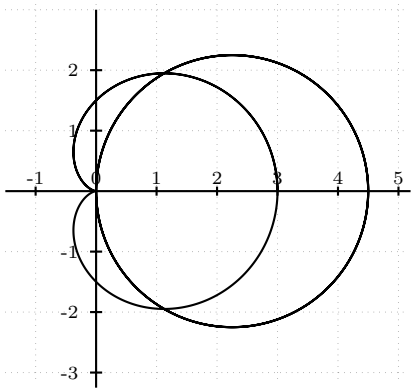


III)



IV)



25. The area between the spirals $r = \theta$ and $r = 2\theta$, $0 \le \theta \le \pi/2$ is

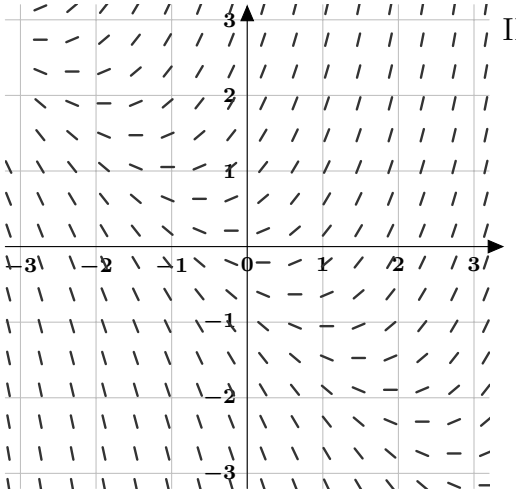
(A) $\dfrac{\pi^2}{4}$ (B) $\pi^2 - 2$ (C) $\dfrac{\pi^3}{16}$ (D) $\pi^3 + 4\pi$ (E) $\dfrac{\pi^4}{8}$

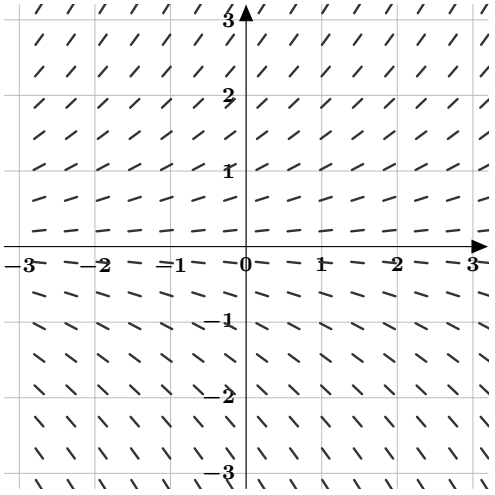
26. Which is true? The point whose Cartesian coordinates are $(1, \sqrt{3})$ has polar coordinates

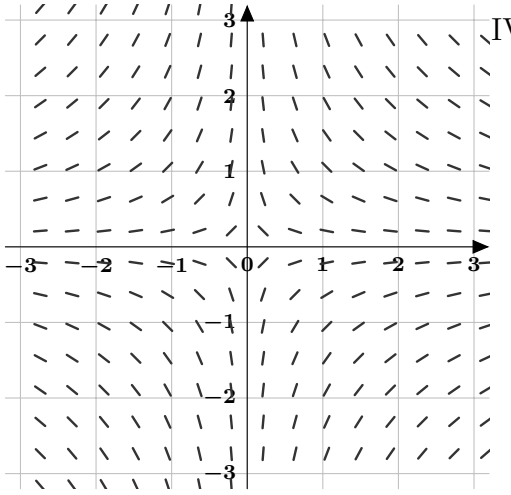
(I) $(2, \pi/3)$ **(II)** $(2, -5\pi/3)$ **(III)** $(2, 4\pi/3)$

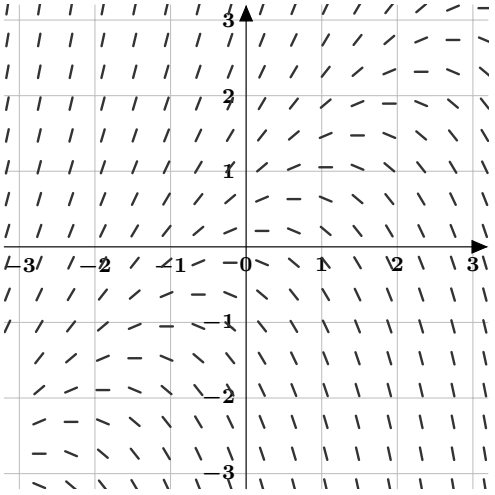
(A) Only **I** (B) Only **II** (C) Only **III** (D) Only **I and II** (E) All are true

27. Math the following slope fields with the correspoding differential equations given below.

I) 

II) 

III) 

IV) 

(a) $\dfrac{dy}{dx} = \dfrac{y}{2}$

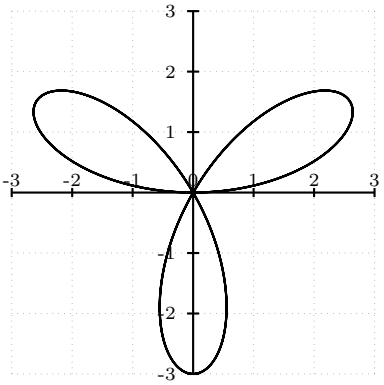
(b) $\dfrac{dy}{dx} = y - x$

(c) $\dfrac{dy}{dx} = x + y$

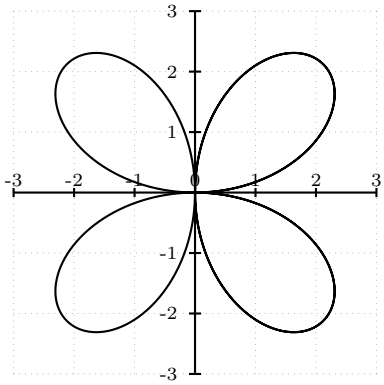
(d) $\dfrac{dy}{dx} = \dfrac{-y}{x}$

28. Select the correct graph of $r = -3\cos 2\theta$.

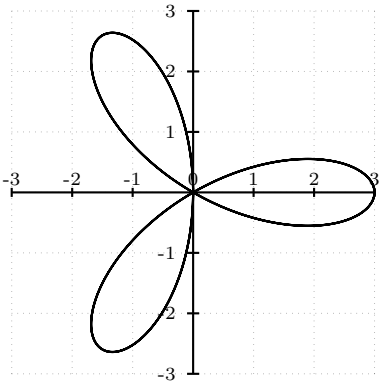
(A)



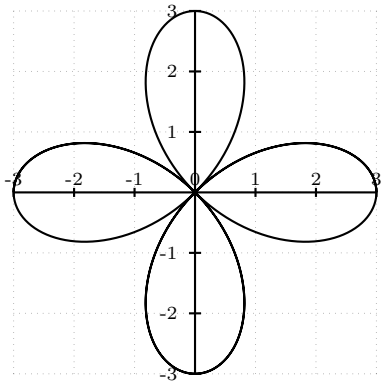
(C)



(B)



(D)



29. The derivative of a function g is $g'(x) = \sqrt{\sec^2 x \tan^2 x - 1}$. What is the length of the curve $y = g(x)$ on the interval $0 \le x \le \pi/4$.

(A) $\sqrt{2} - 1$ (B) $\dfrac{\sqrt{2}}{2} - 1$ (C) $\dfrac{\sqrt{2}}{2}$ (D) $\sqrt{2}$ (E) 1

30. Use Euler's Method with step size $h = 0.5$ to estimate the value of $y(1)$, where y is the solution of the initial value problem:

$$y' = x + y \quad \text{and} \quad y(0) = 1$$

(A) 1 (B) $\dfrac{3}{2}$ (C) $\dfrac{1+\sqrt{5}}{2}$ (D) 2 (E) $\dfrac{5}{2}$

31. A state game commission releases **40** elk into a game refuge. Assume the elk population, $\boldsymbol{P}$, grows according to the following logistic model with a growth constant $\boldsymbol{k = \ln(11/9)}$ per year:

$$\frac{\boldsymbol{dP}}{\boldsymbol{dt}} = \boldsymbol{kP}\left(\mathbf{1} - \frac{\boldsymbol{P}}{\mathbf{4000}}\right)$$

At what time $\boldsymbol{t}$ is the population of elk growing the fastest?

(A) **15** years (B) **23** years (C) **9** years (D) **35** years (E) **5** years