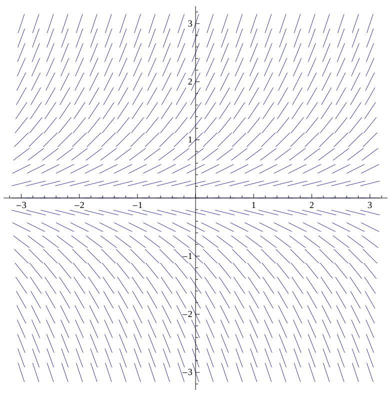
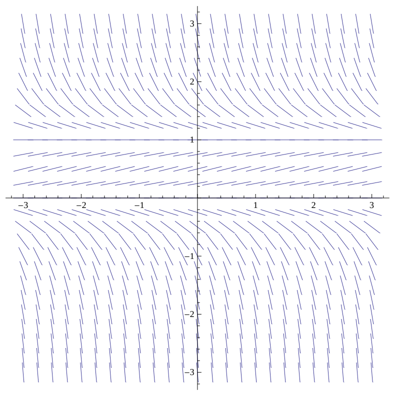
MATH 2300 – review problems for Exam 3, part 2

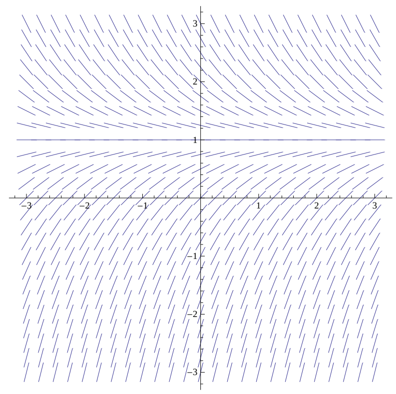
1. Shown below are the slope fields of three differential equations, “A”, “B”, and “C”. For each slope field, the axes intersect at the origin.



A



B



C

For each of the following functions, indicate which, if any, of the differential equations, “A”, “B”, and “C” it could be the solution of. Note that any of the functions could be a solution to zero, one, or more than one of the differential equations. If a function is a solution to none of the differential equations clearly write ”None” as your answer.

(a) $y = 0$
A

(b) $y = 1$
B and C

(c) $y = 1 + ke^x$
None

2. Use Euler’s method with step size $h = 1$ to construct a table of the first three approximate values for the solution of the or the initial-value problem

$$\frac{dy}{dx} = x^2 y - x, y(0) = 2.$$

n	x_n	y_n
0	0	2
1	1	$2 + 1(0^2 \times 2 - 0) = 2$
2	2	$2 + 1(1^2 \times 2 - 1) = 3$
3	3	$3 + 1(2^2 \times 3 - 2) = 13$

3. Find all equilibrium solutions to $\frac{dy}{dx} = \ln(y^2 + 2y + 1)$.
$\frac{dy}{dx} = 0$ when $y^2 + 2y + 1 = 1$ or $y(y + 2) = 0$. Therefore the equilibrium solutions are $y = 0$ and $y = -2$.

4. Verify that $y = -t\cos t - t$ is a solution of the initial-value problem

$$t\frac{dy}{dt} = y + t^2 \sin t \qquad y(\pi) = 0$$

Since $\frac{dy}{dt} = -\cos t + t\sin t - 1$ then

$$t\frac{dy}{dt} = -t\cos t + t^2\sin t - t = (-t\cos t - t) + t^2 \sin t = y + t^2 \sin t.$$

5. Suppose $Q = Ce^{kt}$ satisifies the differential equation

$$\frac{dQ}{dt} = -0.03Q$$

What (if anything) can be determined about the values of k and C?
The differential equation is separable with general solution $Q = Ce^{-0.03t}$ for some constant C. Therefore $k = -0.03$, but we can not determine C without an initial value.

6. Solve (make sure to write your final answer in the form $y = f(x)$ where f is a function of x):

   (a) $\frac{dy}{dx} = -\frac{x}{y^2}$, $y(2) = 1$
   $y = \sqrt[3]{7 - \frac{3}{2}x^2}$

   (b) $y \cdot y' = x(1 + y^2)$, $y(1) = 2$
   $y = \sqrt{5e^{x^2-1} - 1}$

   (c) $y' = y\cos(x)$, $y(0) = 3$
   $y = 3e^{\sin x}$

   (d) $(x^3 + 1)\frac{dy}{dx} - 3x^2 = 0$, $y(1) = \ln 2$
   $y = \ln|x^3 + 1|$

   (e) $\frac{1}{e^{y^3+1}}\frac{dy}{dx} - \frac{1}{3y^2} = 0$, $y(1) = e^2$
   $y = -\sqrt[3]{1 + \ln\left(1 + e^{-e^6-1} - x\right)}$

7. Use implicit differentiation to show that $x^2 + y^2 = r^2$ is a solution to the differential equation $\frac{dy}{dx} = -\frac{x}{y}$
Differentiate: $2x + 2y\frac{dy}{dx} = 0$, solving gives $\frac{dy}{dx} = -y/x$

8. For what values of C and n (if any) is $y = Cx^n$ a solution to the differential equation:

$$x\frac{dy}{dx} - 3y = 0$$

$n = 3$, C can be anything
If the solution satisfies $y = 40$ when $x = 2$, what more (if anything) can you say about C and n?
$C = 5$

9. Do textbook problems p. 498 number 9, p. 506 number 13, and p. 507 number 23.