1. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{3^{n+1}} =$

A) $\frac{2}{3}$ ✓

B) $\frac{1}{2}$

C) $\frac{2}{9}$

D) $\frac{3}{4}$

E) $\frac{4}{3}$

2. Some of the following four statements concerning a series $\displaystyle\sum_{n=1}^{\infty} b_n$ must be false, no matter what the series is. Which?

I. The sequence $\{b_n\}$ and the series $\displaystyle\sum_{n=1}^{\infty} b_n$ both converge.

II. The sequence $\{b_n\}$ converges but the series $\displaystyle\sum_{n=1}^{\infty} b_n$ diverges.

III. The sequence $\{b_n\}$ diverges but the series $\displaystyle\sum_{n=1}^{\infty} b_n$ converges.

IV. The sequence $\{b_n\}$ and the series $\displaystyle\sum_{n=1}^{\infty} b_n$ both diverge.

A) I must be false.

B) II must be false.

C) III must be false. ✓

D) IV must be false.

E) Both II and III must be false.

3. $\lim\limits_{n\to\infty} \dfrac{n\sin n}{n^2+n+2}$

A) 1

B) 0 ✓

C) -1

D) $\dfrac{1}{4}$

E) The sequence diverges.

4. Which statement is true, concerning the series

$$(1)\quad \sum_{n=1}^{\infty} \frac{1}{n+2} \qquad\qquad (2)\quad \sum_{n=1}^{\infty} \frac{n^2}{2n^3-1}$$

A) Both converge.

B) (1) converges, (2) diverges.

C) (1) converges, (2) diverges.

D) Both diverge. ✓

E) None of A, B, C, D is true.

5. The first Taylor polynomial of $f(x) = \dfrac{4}{\sqrt{5-x}}$ about $a = 1$ is?

A) $2 + \dfrac{x-1}{4}$ ✓

B) $4 + \dfrac{x+1}{\sqrt{5}}$

C) $2 - \dfrac{x+1}{\sqrt{5}}$

D) $2 + \dfrac{x-1}{2}$

E) $1 + \dfrac{x-1}{\sqrt{5}}$

6. **Which is true for the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$?**

I. If $a_n \geq 0$ for all n and $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

II. If $b_n \geq a_n \geq 0$ for all n and $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ also diverges.

III. If $a_n \geq 0$ for all n then $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.

A) Only I.

B) Only II. ✓

C) Both I and II.

D) All are false.

E) Both II and III.

7. **Use the Maclaurin series generated by $f(x) = \cos(x^2)$ to compute $\int_0^{1/2} \cos(x^2)\,dx$.**

A) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)2^{n+2}(2n)!}$

B) $\sum_{n=0}^{\infty} \frac{1}{(4n+1)4^n(2n+1)!}$

C) $\sum_{n=0}^{\infty} \frac{2^n}{(4n+4)(2n)!}$

D) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(8n+2)16^n(2n)!}$ ✓

E) $\sum_{n=0}^{\infty} \frac{1}{(2n+1)2^{n+1}(2n+1)!}$

8. **Let $T_3(x)$ denote the third degree Taylor polynomial, centered at 0, of $f(x) = e^{-x}$. According to Taylor's Inequality, $T_3(1)$ approximated e^{-1} with an error $\leq$**

A) $\frac{1}{64}$

B) $\frac{1}{48}$

C) $\frac{1}{36}$

D) $\frac{1}{32}$

E) $\frac{1}{24}$ ✓

9. **The interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{2^n(x-1)^n}{n}$ is**

A) $[1/2, 3/2)$ ✓

B) $(-1, 3]$

C) $[1/2, 3]$

D) $[1/2, 3]$

E) $(1/2, 3/2)$

10. **We approximate $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n(2n-1)}$ by its N-th partial sum. What is the smallest N for which the Alternating Series Error Estimate guarantees that the error will be $< 10^{-2}$?**

A) 3

B) 4

C) 5 ✓

D) 6

E) 8

11. **Is the series** $\displaystyle\sum_{n=0}^{\infty} \sqrt{\frac{1}{2^n} + \frac{3}{4^n}}$ **convergent or divergent?**

A) It is convergent by comparison with $\displaystyle\sum_{n=0}^{\infty} \frac{1}{(\sqrt{2})^n}$ ✓

B) It is divergent by comparison with $\displaystyle\sum_{n=0}^{\infty} \frac{1}{(\sqrt{2})^n}$

C) It is divergent by limit comparison with $\displaystyle\sum_{n=0}^{\infty} \frac{1}{(\sqrt{2})^n}$

D) It is convergent by comparison with $\displaystyle\sum_{n=0}^{\infty} \frac{\sqrt{3}}{2^n}$

E) None of the above answers is correct.

12. **Suppose the power series** $\displaystyle\sum_{n=1}^{\infty} c_n x^n$ **converges when** $x = -3$ **and diverges when** $x = 4$.

Which of the following is(are) always true?

I. $\displaystyle\sum_{n=1}^{\infty} c_n(-2)^n$ **converges.** **II.** $\displaystyle\sum_{n=1}^{\infty} c_n(-5)^n$ **converges.** **III.** $\displaystyle\sum_{n=1}^{\infty} c_n(2.5)^n$ **converges.**

A) I and II only.

B) I and III only. ✓

C) II only.

D) I, II, and III

E) None of the above answers is correct.

13. **Find the coefficient of x^{12} in the Maclaurin series for $f(x) = \sin\left(\frac{x^4}{2}\right)$.**

A) 0

B) $-\frac{1}{24}$

C) $\frac{1}{12}$

D) $-\frac{1}{48}$ ✓

E) $\frac{1}{18}$

14. **Which of the following is true?**

I. The series $\sum_{n=2}^{\infty} (-1)^n \frac{1}{n \ln n}$ **converges conditionally?**

II. The series $\sum_{n=2}^{\infty} (-1)^n \frac{1}{n(\ln n)^2}$ **converges absolutely?**

III. The series $\sum_{n=1}^{\infty} (-1)^n \frac{\arctan n}{n^3}$ **converges absolutely?**

A) I and II only.

B) I and III only.

C) II only.

D) I, II, and III are true. ✓

E) None.

15. **Evaluate the indefinite integral** $\int \frac{x}{1+x^3}\,dx$ **as a power series.**

A) $\sum_{n=0}^{\infty} x^{3n} + C$

B) $\sum_{n=0}^{\infty} (-1)^n x^{3n+1} + C$

C) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{3n+2}}{(3n+2)} + C$ ✓

D) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{3n+1}}{(3n+1)} + C$

E) $\sum_{n=0}^{\infty} \frac{x^{3n}}{3n} + C$

16. **For what value(s) of** p **does the series** $\sum_{n=1}^{\infty} \frac{n^{2p}+1}{\sqrt{n+2}}$ **converge?**

A) $p < -\frac{1}{2}$

B) $p \geq -\frac{1}{2}$

C) $p < 0$

D) $p \geq 0$

E) **No values.** ✓

17. **Which of the following is true?**

I. If $\dfrac{1}{\ln n} \leq a_n$ **for** $n \geq 2$**, then** $\displaystyle\sum_{n=2}^{\infty} a_n$ **diverges.**

II. The alternating series $\displaystyle\sum_{n=1}^{\infty} (-1)^n \sqrt{\frac{n+1}{4n+1}}$ **converges.**

III. If $\dfrac{1}{n} \leq b_n \leq \dfrac{1}{\sqrt{n}}$ **and** $b_n \geq b_{n+1}$**, then** $\displaystyle\sum_{n=1}^{\infty} (-1)^n b_n$ **is conditionally convergent.**

A) I and II only.

B) II and III only.

C) I and III only. ✓

D) I only.

E) III only.

18. **We would like to estimate the value of the series** $s = \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+3}$ **by its** N**-th partial sum,** s_N**, with an error less than** 0.01**, i.e.,** $|s - s_N| < 0.01$**, using the Estimation Theorem for Alternating Series. What is the smallest value of** N **that gives us the estimate within the required error amount?**

A) 7

B) 8

C) 9 ✓

D) 10

E) 11

19. **Let** $f(x) = \sum_{n=1}^{\infty} \frac{3n}{(n+1)^2}(x-1)^n$. **Then** $f'''(1) =$

A) 10

B) $\frac{14}{5}$

C) $\frac{13}{6}$

D) $\frac{27}{8}$ ✓

E) $\frac{1}{9}$

20. $\lim_{n\to\infty} \frac{\sqrt{2n^2+3(-1)^n}}{n+4} =$

A) $\sqrt{2}$ ✓

B) 2

C) 1

D) 0

E) ∞

21. **Which of the following series**

$$\text{(I)} \quad \sum_{n=1}^{\infty}(-1)^n\sqrt{\frac{2n+3}{n+1}} \qquad \text{(II)} \quad \sum_{n=1}^{\infty}\frac{\sqrt{n}}{2^n} \qquad \text{(III)} \quad \sum_{n=2}^{\infty}\frac{1}{n(\ln n)^2}$$

converge?

A) II and III only. ✓

B) II only

C) I and II only.

D) None

E) I, II, and III.

22. **Which of the following power series represents xe^{-x^2}?**

A) $-\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$

B) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$

C) $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$

D) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{n!}$ ✓

E) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{n^2+1}}{(n^2+1)!}$

23. **The series** $\sum_{n=1}^{\infty} \left(\sin\left(\frac{1}{n^2}\right)\right)\left(e^{1/n}\right)$

A) Diverges by the Integral Test.

B) Diverges by Comparison Test to the harmonic series.

C) Diverges by the Test for Divergence.

D) Converges by the Ratio Test.

E) Converges by the Limit Comparison Test to $\sum_{n=1}^{\infty} \frac{1}{n^2}$ ✓

24. Let $f(x)$ be a function defined for $x \geq 1$, such that $\dfrac{1}{\sqrt{x}} \leq f(x) \leq 1$, for all $x \geq 1$. What can be said about the series

$$\text{I. } \sum_{n=1}^{\infty} \frac{f(n)}{\sqrt{n}} \qquad\qquad \text{II. } \sum_{n=1}^{\infty} \frac{f(n)}{n^2}$$

A) Both converge.

B) I diverges and II converges. ✓

C) I converges and II diverges.

D) Both diverge.

E) II converges, but I might converge or diverge.

25. Use the limit comparison test to detrmine the if the series below converge or diverge.

$$\text{I. } \sum_{n=1}^{\infty} \ln\left(1+\frac{1}{n}\right) \qquad\qquad \text{II. } \sum_{n=1}^{\infty} \ln\left(1+\frac{1}{n^2}\right)$$

A) Both converge.

B) I diverges and II converges. ✓

C) I converges and II diverges.

D) Both diverge.

E) II converges, but I might converge or diverge.

26. **Find the interval of convergence of the power series** $\sum_{n=1}^{\infty} (-1)^n \frac{n^4(x-1)^n}{3^n(n^5+2)}$

A) $(2, 4)$

B) $[0, 3)$

C) $(-2, 4]$ ✓

D) $(0, 3]$

E) $[-2, 4)$

27. $\sum_{n=3}^{\infty} \left(\frac{1}{n+2} - \frac{1}{n+3} \right) =$

A) $\frac{1}{2}$

B) $\frac{1}{3}$

C) $\frac{1}{4}$

D) 1

E) $\frac{1}{5}$ ✓

28. **The integral** $\int_0^1 \frac{e^x}{\sqrt{x}}\,dx$ **is equal to**

A) $\sum_{n=0}^{\infty} \frac{2}{n!(2n+1)}$ ✓

B) $\sum_{n=0}^{\infty} \frac{1}{n!(n+1)}$

C) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{(n+1)\sqrt{n}}$

D) $\sum_{n=0}^{\infty} \frac{1}{(n+1)!}$

E) $\sum_{n=0}^{\infty} \frac{e^n}{\sqrt{n+1}}$

29. $\sum_{n=1}^{\infty} \frac{2^n + 3^{n+1}}{5^n} =$

A) $\frac{12}{7}$

B) 8

C) $\frac{18}{5}$

D) $\frac{31}{6}$ ✓

E) $\frac{27}{18}$

30. **Let $a_n = \dfrac{\cos(1/n)}{2n+1}$. Which of the following is true?**

A) The sequence $\{a_n\}$ is divergent and the series $\sum_{n=1}^{\infty} a_n$ is divergent.

B) The sequence $\{a_n\}$ is convergent and the series $\sum_{n=1}^{\infty} a_n$ is divergent. ✓

C) The sequence $\{a_n\}$ is divergent and the series $\sum_{n=1}^{\infty} a_n$ is convergent.

D) The sequence $\{a_n\}$ is convergent and the series $\sum_{n=1}^{\infty} a_n$ is convergent.

E) None of A, B, C, or D are true.

31. **Evaluate $\lim_{x\to 0} \dfrac{x^6 - 12x^2 + 24\arctan(x^2/2)}{x^{10}}$**

A) $\dfrac{3}{16}$

B) $\dfrac{3}{20}$ ✓

C) $\dfrac{3}{10}$

D) 0

E) The limit does not exist.

32. **Find the sum of the series** $\sum_{n=1}^{\infty} \frac{n}{3^{n-1}}$

A) 3

B) 2

C) $\frac{7}{3}$

D) $\frac{9}{4}$ ✓

E) $\frac{3}{2}$

33. **If the Maclaurin series of a function** $f(x)$ **is** $\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{3n(n+6)}$. **Then** $f^{(6)}(0) =$

A) $\frac{16}{3}$

B) $\frac{20}{3}$

C) $\frac{10}{3}$ ✓

D) 0

E) None of these.

34. **Use Taylor's Inequality to determine the maximum error in the approximation of $\cos(1)$ by $T_4(1)$ if $T_4(1)$ is the fourth Taylor polynomial of $f(x) = \cos x$ centered at 0.**

A) $\frac{1}{120}$ ✓

B) $\frac{13}{120}$

C) $\frac{1}{24}$

D) $\frac{13}{24}$

E) $\frac{\pi}{8}$

35. **Which of the following is the Maclaurin series for the function $f(x) = \ln \frac{1-x}{1+x}$**

A) $-2\left[x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \cdots\right]$ ✓

B) $\frac{1}{2}\left[x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots\right]$

C) $-\left[x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \cdots\right]$

D) $\frac{1}{2}\left[x + \frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \cdots\right]$

E) $2\left[x - \frac{x^2}{2} + \frac{x^4}{4} - \frac{x^6}{6} + \cdots\right]$

36. **The series** $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{\arctan n}{n^2+1}$ **is**

A) absolutely convergent. ✓

B) conditionally convergent.

C) divergent since $\displaystyle\lim_{n\to\infty}(-1)^n\frac{\arctan n}{n^2+1}\neq 0$

D) divergent even though $\displaystyle\lim_{n\to\infty}(-1)^n\frac{\arctan n}{n^2+1}=0$

E) divergent by the ratio test.

37. **Use a Maclaurin series and the Estimation Theorem for Alternating Series to approximate** $\sin\left(\frac{1}{2}\right)$ **using the fewest number of terms necessary so that the error is less than** 0.001

A) $\frac{1}{2}$

B) $\frac{23}{48}$ ✓

C) $\frac{3}{4}$

D) $\frac{33}{40}$

E) $\frac{41}{60}$

38. **In the Taylor series expansion for** $f(x) = \dfrac{x-1}{x-2}$ **about** $a = 1$**, the coefficient of** $(x-1)^{10}$ **is**

A) -2

B) -1

C) 0

D) 1 ✓

E) 2

39. **Which of the following series converge?**

I. $\displaystyle\sum_{n=2}^{\infty} \frac{\sqrt{n}+1}{n^2}$ II. $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{n!}$ III. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{5^n-2}$ IV. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[4]{n}}$

A) III only.

B) III and IV only.

C) All four.

D) I, III, and IV only. ✓

E) II, III, and IV only.

40. **Find the interval of convergence of the power series** $\displaystyle\sum_{n=1}^{\infty} \frac{10^n}{n!}(x-1)^n$

A) $(0,2)$

B) $[0,2)$

C) $(9,11)$

D) $[9,11)$

E) $(-\infty,\infty)$ ✓

41. **Evaluate** $\lim\limits_{x\to 0} \dfrac{\cos(x^2) - 1 + (x^4/2)}{x^8}$

A) $\dfrac{1}{2}$

B) $\dfrac{1}{8}$

C) $\dfrac{1}{6}$

D) $\dfrac{1}{120}$

E) $\dfrac{1}{24}$ ✓

42. **The first three non-zero terms of the McLaurin series of** $f(x) = x\ln(1+x^2)$ **are**

A) $x - \dfrac{1}{3}x^3 + \dfrac{1}{5}x^5$

B) $x^2 - \dfrac{1}{2}x^4 + \dfrac{1}{3}x^6$

C) $x^2 - \dfrac{1}{2}x^4 + \dfrac{1}{6}x^6$

D) $x^3 - \dfrac{1}{2}x^5 + \dfrac{1}{3}x^7$ ✓

E) $x^3 - \dfrac{1}{5}x^5 + \dfrac{1}{7}x^7$

43. If $\sum_{n=1}^{\infty} |a_n|$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

A) TRUE ✓ B) FALSE

44. If $a_n > 0$ and $\lim_{n\to\infty} n a_n = 2$, then $\sum_{n=1}^{\infty} a_n$ diverges.

A) TRUE ✓ B) FALSE

45. If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{2}$, then $\sum_{n=1}^{\infty} a_n$ converges.

A) TRUE ✓ B) FALSE

46. $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$ converges conditionally.

A) TRUE B) FALSE ✓

47. $\sum_{n=1}^{\infty} a_n = \lim_{N\to\infty} \left(\sum_{n=1}^{N} a_n\right)$.

A) TRUE ✓ B) FALSE

48. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ converges.

A) TRUE B) FALSE ✓

49. $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$ converges.

A) TRUE B) FALSE ✓

50. **If $f(x) = 4 + x - x^2 + x^3 - x^4 + \cdots$, then $f'''(0) = 6$.**

A) TRUE ✓ **B) FALSE**

51. **If $f(x) = \sum_{n=1}^{\infty} \frac{n}{(n+1)!} x^n$, then $f^{(5)}(0) = \frac{1}{6}$.**

A) TRUE **B) FALSE ✓**

52. **The radius of convergence of the power series $\sum_{n=1}^{\infty} 2^n x^n$ is 2.**

A) TRUE **B) FALSE ✓**

53. **The radius of convergence of $\sum_{n=1}^{\infty} 2^n x^n$ is equal to the radius of convergence of $\sum_{n=1}^{\infty} \sqrt{n}\, 2^n x^n$.**

A) TRUE ✓ **B) FALSE**

54. **If $0 \leq a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.**

A) TRUE ✓ **B) FALSE**

55. **If $0 \leq b_n \leq a_n$ and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges.**

A) TRUE ✓ **B) FALSE**