1. **The equations of one dimensional motion**
   Let's summarize how we can describe the one dimensional motion of an object with equations. First, we have to set up a coordinate system, in the one dimensional case simply a coordinate axis. Setting up a coordinate axis means fixing the point 0 to some point in the line of the motion, choosing a unit and choosing the positive direction. Once we have a coordinate axis, we can talk about the following functions that describe the motion of the object:

   - $s(t)$: position of the object as a function of time,
   - $v(t) = s'(t) = \dfrac{ds}{dt}$: velocity function,
   - $a(t) = v'(t) = \dfrac{dv}{dt} = s''(t) = \dfrac{d^2s}{dt^2}$: acceleration function.

   As we can see, if we are given the position function $s(t)$, it is easy to find the velocity $v(t)$, and the acceleration $a(t)$ functions: we just have to take the first and second derivatives of $s(t)$. On the other hand to find the position function given the velocity or to find the velocity function given the acceleration, we have to solve differential equations and find antiderivatives. The results are:

   - $v(t) = v(t_0) + \displaystyle\int_{t_0}^{t} a(u)\,du$,
   - $s(t) = s(t_0) + \displaystyle\int_{t_0}^{t} v(u)\,du$.

   If we denote the initial velocity by $v_0 = v(0)$ and the initial position by $s_0 = s(0)$, then these equations have the form:

   - $v(t) = v_0 + \displaystyle\int_{0}^{t} a(u)\,du$,
   - $s(t) = s_0 + \displaystyle\int_{0}^{t} v(u)\,du$.

2. **Equations of free fall**
   Let's specialize these equations to the case of one dimensional free fall. Typically, the coordinate axis that we use is in a vertical position, the positive direction pointing upward and the zero fixed to the ground level. However, this is not necessary, for example if we drop an object from the top of a building, we could fix the zero to the top of the building and make the positive direction downward. From now on we assume that the zero is fixed to ground level and the positive direction is upward. We know that the acceleration of an object in free fall is the constant $g \approx 9.8m/s^2 \approx 32ft/s^2$. Then applying the above integral formulas we get the following equations:

- $a(t) = -g$,
- $v(t) = v_0 - gt$,
- $s(t) = s_0 + v_0 t - g\dfrac{t^2}{2}$.

3. **Newton's law of motion**
   Newton's law of motion tells us how forces acting on some object affect the motion of the object. It states that $F = ma$, where F is the sum of all forces acting on the object, m is the mass of the object and a is the acceleration. In particular, if the net force acting on the object is zero, then the object's acceleration is also zero. An object with zero acceleration has constant velocity, but not necessarily a constant position. Thus an object under no influence from other physical entities will move in a straight line with constant velocity. Of course, this constant velocity can be zero, in which case the object does not move.

   Remark. Newton's law of motion ($F = ma$) is only true in certain coordinate systems. Such a coordinate system is called inertial frame of reference. If you ask a physicist what an inertial frame of reference is they will probably say that it is a coordinate system in which Newton's law of motion is true. This argument seems a bit circular and might be confusing. Ideally, an inertial frame would be a coordinate frame that is not moving and not rotating in an absolute sense. The problem with this is that every motion is relative, and when we use a coordinate frame, we have to fix it to something. For example, an accelerating train would definitely be a bad choice for an inertial frame. In fact for each application we can choose a coordinate system that can be considered to be an inertial frame for the given application. For certain applications a corner of a room can be a good reference point, for others it should be the center of the Earth, and again for others the center of the Sun.

4. **Force of gravity**
   Physical experiments tell us that any two objects act upon each other by an attracting force that is due to their mass. There is an attracting force between the Sun and the Earth, the Earth and the Moon, the Moon and the astronaut walking on it. The equation describing the gravitational force is

   $$F = \frac{Gm_1m_2}{r^2},$$

   where G is the gravitational constant (its value is $\approx 6.674 \times 10^{-11}\ \mathrm{N}\,(\mathrm{m}/\mathrm{kg})^2$), m_1 and m_2 are the masses of the two objects and r is the distance between the objects, more precisely, the distance between their centers of mass.

   Let's apply this law to understand free fall close to the surface of the Earth. In this case the force due to gravity acting on some object is $F = GMm/R^2$, where M is the mass of the Earth, m is the mass of the object and R is the radius of the Earth. Of course, we are using an approximation here, in reality the distance between the Earth and the object changes, but this change is negligable compared to the radius of the Earth. Now let's plug this formula for the force into Newton's law:

$$-\frac{GMm}{R^2} = ma,$$

where the negative sign comes from our choice of the positive direction being upward. After simplification we get

$$a = -\frac{GM}{R^2}.$$

We see that this is a constant and this value does not depend on the mass of the object. This was the great discovery of Galilei (the book calls him Galileo, but that's actually his first name), that the acceleration of an object in free fall (neglecting air resistance) is independent of the mass of the object.

5. **Example.**

   An object is dropped from a 400-foot tower. When does it hit the ground and how fast is it going at the time of the impact?

   Solution. To be able to answer these questions we have to find the equations for the velocity and the position. The initial velocity (v_0) is zero, because the object is simply released from the tower. The initial position (s_0) is 400 feet, the height of the tower. Using our formulas for free fall, we have

$$v(t) = v_0 - gt = -32t, \qquad s(t) = s_0 + v_0 t - g\frac{t^2}{2} = 400 - 16t^2,$$

   where we used $g = 32ft/s^2$, as the height was given in feet. The object hits the ground when the position is zero, so we have to solve

$$s(t) = 0 :$$

$$400 - 16t^2 = 0,$$

$$t^2 = 25,$$

$$t = 5.$$

   So the object hits the ground at $t = 5\,sec$. To find the velocity at that moment we just have to plug 5 into the velocity function: $v(5) = -32 \cdot 5 = -160\ ft/sec$.