1. Section 4.2 of HH - Optimization
   (a) Global Maxima and Minima. f has a global minimum/maximum at p if $f(p)$ is less than/greater than or equal to all values of f.
   (b) For example, the function $f(x) = x^2$ has a global minimum at $x = 0$.
   (c) For consideration, the function $f(x) = x$ has *no* global maximum or minimum.

2. Global extrema on a closed interval. On a closed interval $[a, b]$, we can *guarantee* the existence of a global max or min.
   - For a continuous function f on a closed interval $a \leq x \leq b$,
   - Find the critical points of f on the interval.
   - Evaluate the function at the critical points *and* at the endpoints a and b.
   - Choose the largest/smallest of the function values at the endpoints/critical points.

3. Example

   Let $f(x) = x^2$ on $[-1, 2]$.

   First, we find the critical points. $f'(x) = 2x$, so we have one critical point at $x = 0$.

   Then we check $f(x)$ at the critical point and the endpoints.

$$\begin{aligned} f(-1) &= 1 \\ f(0) &= 0 \\ f(2) &= 4 \end{aligned}$$

   Thus, f has a global minimum at $x = 0$ and a global maximum at $x = 2$. (Note that $f(x)$ has a global maximum because of the restriction to $[-1, 2]$. If we considered $f(x)$ on all real numbers, it would have no maximum).

4. Global extrema on an open interval (or all real numbers). In this case, we cannot guarantee the existence of global extrema.

5. Example

   Let $g(x) = x^3$ on $(-1, 1)$.

   There is a critical point here, but g has no global max or min. One way to think about this is that $-1 < g(x) < 1$ for all x-values in the domain, but g is never equal to ± 1.

6. Example

   Let $h(t) = te^{-t}$ on $(0, \infty)$.

   Then $h'(t) = e^{-t}(1-t)$, so we have one critical point at $t = 1$ (use the first derivative test to check that this is a local maximum). Furthermore, this is the only critical point, so our intuition would tell us that the graph of h won't "turn" back up. Thus, h has a global max at $t = 1$.

   In fact, this is an instance of a nice theorem:

   Theorem. If a function is continuous and has exactly one critical point, and that critical point is a local min/max, then it is a global min/max.

7. In general, it is useful to look at the graph of a function, because it will give some idea about the behavior of the graph as $x \to \pm\infty$. For example, the graph of $\arctan x$ is bounded: $-\frac{\pi}{2} < \arctan x < \frac{\pi}{2}$, but never attains those values, so there is no max or min.