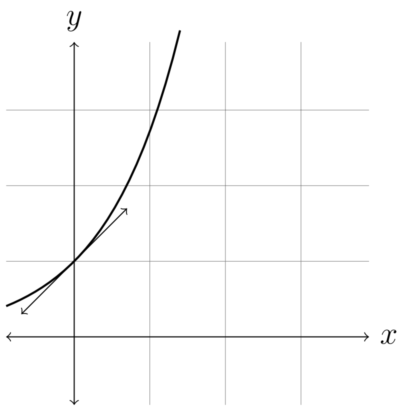
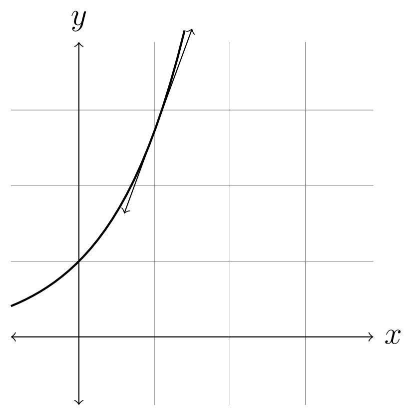
The purpose of this section is to learn and to use the derivative formulas for exponential functions.

Bottom line: if $f(x) = e^x$, $f'(x) = e^x$.

Wow, it is equal to its own derivative! Only this function, or a constant multiple of it, has this amazing property. The function grows exactly so that its height at any point is the same as its slope.
The picture on the left shows that the height of the function at $x = 0$ is 1, and the slope of the tangent line there is also 1. The picture on the right shows that the height of the function at $x = 1$ is e, and the slope of the tangent line there is also e.





What about exponential functions with bases other than e? The derivative is almost the same as itself, but it's off by a constant:

Bottom line: if $f(x) = a^x$, $f'(x) = (\ln a)a^x$.

The rough idea of how to show these are the correct formulas is given in the text.

Example: Find the derivative of $f(x) = 3e^x$
Use the constant multiple rule:
$f'(x) = 3\frac{d}{dx}(e^x) = 3e^x$.

Example: Find the derivative of $f(x) = x^2 + 2^x$.
It's a sum, so we can differentiate separately and then add (the sum rule from Section 3.1). Notice that the variable is in a different place in the two terms. The first term, x^2, has the variable in the base, so it is a power function. $\frac{d}{dx}(x^2) = 2x$ (Section 3.1). The second term has the variable in the exponent, its derivative is what we learned in this section: $\frac{d}{dx}(2^x) = (\ln 2)2^x$. So $f'(x) = 2x + (\ln 2)2^x$

Example: Find the derivative of $f(x) = e^{x+3}$.
This function is a composition ($x+3$ is the "inside function", e^x is the "outside function"). In Section 3.4 (the chain rule), we'll learn how to handle taking derivatives of compositions. Until then, we can figure out this derivative by rewriting: $f(x) = e^{x+3} = e^3e^x$, a constant times e^x, so we can use the constant multiple rule:
$f'(x) = \frac{d}{dx}(e^{x+3}) = \frac{d}{dx}(e^3e^x) = e^3\frac{d}{dx}(e^x) = e^3e^x = e^{x+3}$.

Example: Find the equation of the tangent line to $f(x) = 5^{2x}$ at $x = 1$.
First we need the formula for the derivative of 5^{2x}. Rewriting is useful here again. By the rules of exponents, $f(x) = 5^{2x} = (5^2)^x = 25^x$. So $f'(x) = (\ln 25)25^x$. We use this formula to find $f'(1) = (\ln 25)25^1 = 25\ln 25$. This is the slope of the tangent line. We also need a point on the tangent line. Substituting $x = 1$ into the function gives the y-value, $f(1) = 5^2 = 25$. So the point $(1, 25)$ lies on the tangent line. Using point/slope form, the equation of the tangent line is

$$y - 25 = 25\ln 25(x - 1).$$

Note that $\ln(25) = \ln(5^2) = 2\ln 5$. So we can simplify our line to

$$y - 25 = 50\ln 5(x - 1).$$

Example: Find the derivative of e^{x^2}.
Unlike the last two examples, the laws of exponents don't allow us to simplify this into the form a^x. So we don't know how to take the derivative of this function yet.