MATH 1300 **Lecture Notes** **Monday, August 26, 2013**

1. Section 1.1 of HH - Functions and Change

   (a) Functions: A *function* is a rule that relates two quantities, x (the independent variable) and y (the dependent variable), such that each input (a value for x) is related to exactly one output (a value for y).

   (b) Rule of Four: In this course we will often represent functions in four different ways: with graphs, with formulas, numerically (in tables) or with words. Almost every concept we learn can be understood and applied to each of these ways of describing functions.

   (c) Domain/Range: The domain of a function is the set of all possible input values of the function. The range of a function is the set of all possible output values.

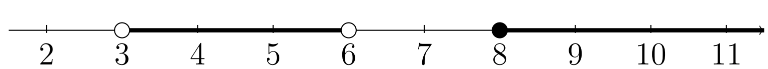
   - If a function is given graphically, the domain and range can be determined directly from the graph by looking at the extent of the function in the x direction (domain) and y direction (range).
   - If a function is defined by a formula, there are a couple of restrictions to the domain that you should already be familiar with. There can be no zeroes in the denominator, and there can be no negatives under a square root (or any even root). In a few sections we will see other kinds of domain restrictions (in log functions and inverse trig functions.)

   (d) Function notation: The dependent variable y is often written in function notation as $f(x)$ ($f(x)$ is the output that is associated with the input x). For example, the function $y = 2x + 4$ would be written using function notation as $f(x) = 2x + 4$. $f(2) = 8$ means that 8 is the y-value associated to the x-value of 2.

   (e) Increasing versus decreasing: Looking at the graph of a function, if it is rising as you move from left to right, we call that function increasing, if it is dropping as you move from left to right, it is decreasing. When we are asked "where" a function is increasing/decreasing, we answer by giving the x-values where the function is increasing/decreasing. Give the answer in interval notation. Recall interval notation is a way of describing intervals on the number line:

   is represented in interval notation as $(3, 10]$, and

   is represented by $(3, 6) \cup [8, \infty)$.

   (f) Proportionality: If y is directly proportional to x, that means $y = kx$, (where k is a constant). y indirectly (or inversely) proportional to x means $y = \frac{k}{x}$.

   (g) Lines: If a function has a constant rate of of change (the output increases/decreases by the same amount when the input increases by one unit).

- slope/intercept form: $y = mx + b$ (where m is the slope and b is the y-intercept). This may be written using function notation as $f(x) = mx + b$.
- point/slope form: $y - y_1 = m(x - x_1)$ (where m is the slope and the point (x_1, y_1) lies on the line)
- Parallel lines have the same slope, while the slopes of perpendicular lines are opposite reciprocals of each other. For example, if a line has slope 3, a perpendicular line would have slope $\frac{-1}{3}$.

2. Examples

(a) What is the domain of the function $f(x) = \frac{2}{\sqrt{2x^2-8}}$?

Solution. $f(x)$ is undefined for x values that yield a 0 in the denominator. The denominator is 0 when

$$\begin{aligned} \sqrt{2x^2 - 8} &= 0 \\ 2x^2 - 8 &= 0 \\ 2x^2 &= 8 \\ x^2 &= 4 \\ x &= \pm 2 \end{aligned}$$

So $x = \pm 2$ is not in the domain of $f(x)$. $f(x)$ is also undefined for x values that yield a negative number under the square root. The argument under the square root is negative when

$$\begin{aligned} 2x^2 - 8 &< 0 \\ 2(x^2 - 4) &< 0 \\ x^2 - 4 &< 0 \end{aligned}$$

To solve quadratic inequalities, we first find the zeros of the quadratic, which in this case, we find the zeros of $x^2 - 4$ to be ± 2. The zeros partition the number line into three regions: $(-\infty, -2), (-2, 2), (2, \infty)$. Testing a point from each region, we see that $x^2 - 4$ is positive on $(-\infty, -2)$ and $(2, \infty)$, and negative on the interval $(-2, 2)$. So $(-2, 2)$ is not in the domain of $f(x)$.

Combining this with our findings from above that ± 2 is not in the domain, we have that $[-2, 2]$ is not in the domain of $f(x)$, so the complement of this set, $(-\infty, -2) \cup (2, \infty)$, is the domain of $f(x)$.

(b) Write the equation of the line which is perpendicular to the line $y = -2x + 4$ and goes through the point $(1, 4)$.

Solution. The easiest way to solve this problem is using point-slope form. Since the line must be perpendicular to a line with slope -2, we know the slope is $\frac{1}{2}$. So the equation of the line is

$$y - 4 = \frac{1}{2}(x - 1).$$

This can be manipulated to slope-intercept form by using algebra to solve for y, in which we obtain

$$y = \frac{1}{2}x + \frac{7}{2}.$$

Either form is acceptable.

(c) Write a function that models the following: attendance at a sporting event is a function of the ticket price. When tickets are free, the event sells out and all $20,000$ seats are filled. For every 10 dollar increase in price, attendence *decreases* by $1,000$ people.

Solution. Since attendence is decreasing a constant amount for every 10 dollar increase, this is describing a linear relationship. The slope is given by the change in the dependent variable (attendence) over the change in the independent variable (ticket price), which is

$$\frac{-1,000 \text{ people}}{10 \text{ dollars}} = -100\frac{\text{people}}{\text{dollar}}.$$

Note that the units of the slope are always the units of the dependent variable over the units of the independent variable. The y-intercept is the attendence when the tickets are 0 dollars (i.e., when tickets are free), so the y-intercept is $20,000$ people. Therefore, if $A(t)$ is the attendence when tickets cost t dollars, the function that models this relationship is

$$A(t) = -100t + 20,000.$$

(d) Suppose P is directly proportional to the square of t with proportionality constant 5. At what value(s) of t does the graph of P intersect the line $y = -5t + 10$?

Solution. Since P is proportional to the square of t with proportionality constant 5, we have that $P = 5t^2$. When two graphs intersect, this just means their y-values are the same, so this happens when $P = y$, or equivalently, when

$$5t^2 = -5t + 10.$$

When solving quadratics, we want to get the quadratic in standard form, so in this case we have

$$5t^2 + 5t - 10 = 0.$$

We can always use the quadratic formula to solve equations in this form, but sometimes, we can also factor:

$$0 = 5t^2 + 5t - 10 = 5(t^2 + t - 2) = 5(t - 1)(t + 2).$$

So either $t - 1 = 0$ or $t + 2 = 0$, so either $t = 1$ or $t = -2$. Therefore, the graph of P intersects the line at $t = 1$ and $t = -2$.