

Review problems for Final Exam
Mathematics 1300, Calculus 1

1. For each part, find dy/dx .

(a) $y = \frac{3x^3 + 2x^2}{\sqrt{x}}$

(b) $y = xe^{2x}$

(c) $y = \frac{1}{2} \sin(-2x)$

(d) $xy = x^2 + y^2$

(e) $y = \int_3^x e^{-t^2} dt$

(f) $y = (-5x + 2)^4$

(g) $y = \int_x^2 3 \sin(t^2) dt$.

(h) $\frac{d}{dx} [\pi^e + 2]$

(i) $\frac{d}{dx} [3^x + x^3]$

(j) $\frac{d}{dt} [\arctan(t) \ln(t/5)]$

(k) $\frac{d}{dy} \left[\sqrt{\frac{y+1}{y+7}} \right]$

(l) Let $y = \int_{2x}^{x^3} \frac{8}{\ln(4t)} dt$, for $x > 0$.

2. Find the most general antiderivative.

(a) $\int \frac{1}{x^3} dx$

(b) $\int \frac{1}{\cos^2 x} dx$

(c) $\int (3e^{2x} + 2 \sin x) dx$

(d) $\int (x+1)^9 dx$

(e) $\int \frac{x^2 + x + 1}{x} dx$

3. Evaluate the following integrals

(a) $\int_0^2 \sqrt{4-y^2} dy$

(b) $\int \frac{t + \sqrt{t} + 1}{t^2} dt$

(c) $\int_1^2 [2^\theta + 1] d\theta$

(d) $\int \sin x \cos^2 x dx$

(e) $\int_0^1 x(x^2 + 1)^4 dx$

4. Let $C(q)$ be the total cost of producing q lawnmowers. Which of the following gives the meaning of $C'(1000)$?

- (a) The cost of producing 1001 lawnmowers.
- (b) The average cost of producing each of the first 1000 lawnmowers.
- (c) The approximate cost of producing the 1001st lawnmower.
- (d) None of the above.

5. Suppose $f(x)$ is continuous on $[-1, 1]$ and differentiable on $(-1, 1)$. Which of the following is true?

- (a) If $f(x)$ has a critical point at $x = 0$ and $f''(x) < 0$, then $f(x)$ has a local minimum at $x = 0$.
- (b) If $f(x)$ has a critical point at $x = 0$ and $f''(x) > 0$, then $f(x)$ has a local minimum at $x = 0$.
- (c) If $f(x)$ has a critical point at $x = 0$, then $f(x)$ has a local minimum or maximum at $x = 0$.
- (d) If $f(x)$ has a critical point at $x = 0$ and $f''(0) = 0$, then $x = 0$ is an inflection point.

6. Suppose that $F(x) = \int_1^x \ln t dt$ for $x > 0$. Which of the following statements is false?

- (a) $F(1) = 0$.
- (b) $F'(e) = 1$.
- (c) $F(x)$ is increasing at $x = 2$.
- (d) $F(x)$ is increasing at $x = \frac{1}{2}$.

7. What are the inflection points of $f(x) = x^5 - 5x^4 - 50$?

8. Solve the differential equation $\frac{dq}{dz} = 2 + \sin(z)$ with initial condition $q(\pi) = 2\pi$.

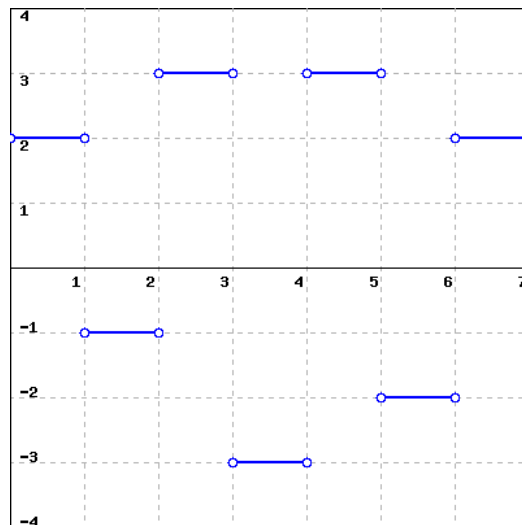
9. Using the left-hand Riemann sum with $n = 4$, approximate $\int_1^9 \frac{1}{x} dx$.

10. Suppose that $f(2) = 4$, and that the table below gives values of f' for x in the interval $[0, 12]$

x	0	2	4	6	8	10	12
$f'(x)$	-19	-21	-25	-28	-29	-28	-25

Estimate $f''(2)$, and estimate $f(8)$.

11. If $H(3) = 1$, $H'(3) = 3$, $F(3) = 5$, $F'(3) = 4$, find $G'(3)$ if $G(w) = F(w)/H(w)$.
12. What is the largest area a rectangle with a perimeter of 40 inches can have?
13. A rocket's height (in feet) is given by $s(t) = 3e^{2t} + 10$, where t is in seconds. How fast is the rocket traveling when it reaches a height of 40 feet?
14. A circular oil spill is growing. Its radius is increasing at a rate of 10 feet per minute. When the radius is 2500 feet, at what rate is the area of the oil spill growing?
15. Assume f' is given by the graph below.



- a) Find the value of the integral: $\int_0^7 f'(x) dx$
- b) What is the second derivative of f , i.e., f'' ?
- c) Suppose f is continuous and that $f(0) = 1$. Sketch an accurate graph of f in the above box (which already contains a graph of f').

16. Represent the finite area enclosed by the two curves $y = x^2$ and $y = \sqrt{x}$ as an integral and evaluate it.
17. You have 80 feet of fencing and want to enclose a rectangular area up against a long, straight wall (using the wall for one side of the enclosure and the fencing for the other three sides of the enclosure). What is the largest area you can enclose?
18. Find the global maximum and minimum for $h(x) = \frac{x+1}{x^2+3}$ on the interval $-1 \leq x \leq 2$.
19. Fill in each of the blanks below with the best possible answer.
- a) If f is differentiable on $a \leq x \leq b$, then there exists a number c , with $a < c < b$, such that

$$f'(c) = \underline{\hspace{2cm}}.$$

b) For any function f , the function whose value at x is given by

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h},$$

is called _____.

20. Evaluate each of the following limits.

a) $\lim_{x \rightarrow 0} \frac{2^{-x} - x}{x^2 + 9}$

b) $\lim_{x \rightarrow \infty} \frac{3x^2 - x^4 - 5x^5}{x^5 - 1}$

c) $\lim_{x \rightarrow 0^-} \frac{|x - 1|}{x - 1}$

d) $\lim_{x \rightarrow 1^-} \frac{|x - 1|}{x - 1}$

e) $\lim_{x \rightarrow \infty} x^2 e^{-x}$

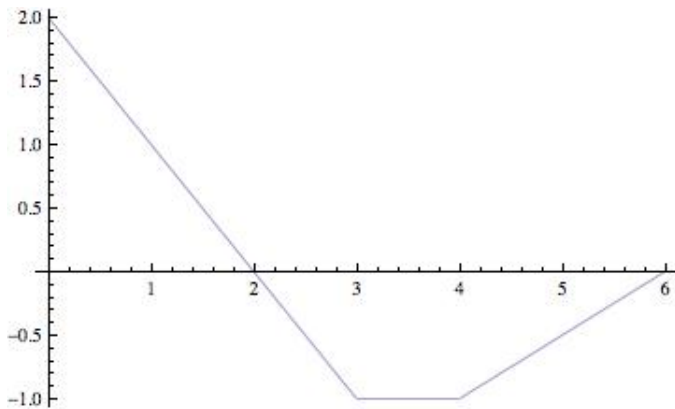
f) $\lim_{x \rightarrow 0^+} x \ln x$

21. Find the equation of the tangent line to the curve $y = \sqrt{25 - x^2}$ where $x = 3$.

22. Let $f(x)$ be the function whose graph is given below. Define

$$F(x) = \int_0^x f(t) dt.$$

Use the graph to fill in the entries in the table below:



x	2	3
$F(x)$		
$F'(x)$		
$F''(x)$		

23. A 10-meter ladder is leaning against the wall of a building. The base of the ladder begins sliding away from the building at a rate of 3 meters per second. How fast is the top of the ladder sliding down the wall when the base of the ladder is 6 meters from the wall?

24. If $f(x)$ is a function having all four of the following properties:

$$\int_0^1 f'(x) dx = 2, \quad \int_1^2 f'(x) dx = 4, \quad \int_2^3 f'(x) dx = 8, \quad f(3) = 16,$$

then determine the following:

(a) $\int_0^3 f'(x) dx$

(b) $f(0)$.

25. Let

$$F(x) = \int_0^{x^2} e^{-4t^2} dt.$$

(a) Find $F'(x)$.

(b) Find $F''(x)$.

(c) Find the x coordinates of all inflection points of $F(x)$.

26. Find the area between the graphs of $f(x) = x + 6$ and $g(x) = x^2$.

27. State the definition of the derivative of $f(x)$ at $x = a$.

28. Find the following derivatives using the definition of derivative (as a limit of difference quotients). Note: For this problem you may use shortcuts (derivative formulas) to **check** your solution. However, to get full credit for this problem you need to use the **definition of the derivative** to find $f'(x)$.

(a) $f(x) = \frac{2}{x^2}$.

(b) $g(x) = \sqrt{x}$.

(c) $h(x) = 3x^2$.

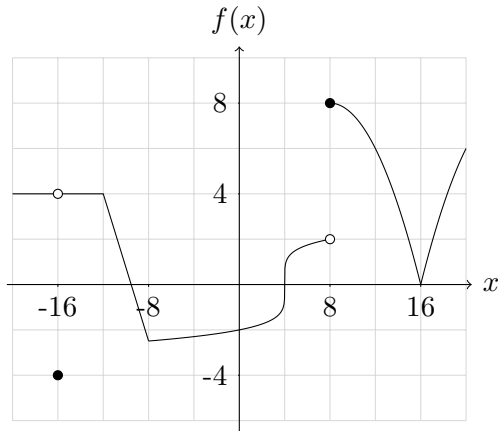
29. Sketch the graph of a continuous, differentiable function f , whose domain is $0 < x < \infty$, given the following information about f , f' , and f'' :

$$\lim_{x \rightarrow 0^+} f(x) = -\infty,$$

$$\begin{array}{lll} f(1) = 0, & f(x) < 0 \text{ for } x < 1, & f(x) > 0 \text{ for } x > 1, \\ f'(2) = 0, & f'(x) > 0 \text{ for } x < 2, & f'(x) < 0 \text{ for } x > 2, \\ f''(3) = 0, & f''(x) < 0 \text{ for } x < 3, & f''(x) > 0 \text{ for } x > 3, \end{array}$$

$$\lim_{x \rightarrow +\infty} f(x) = 0.$$

30. The graph of the function $f(x)$ is given below:



List all x -values at which $f(x)$ is **not** continuous, and list all x -values at which $f(x)$ is **not** differentiable.

31. Assuming that $f(1) = 9$ and $f'(1) = 4$, find

- (a) $g'(1)$, where $g(x) = \sqrt{f(x)}$.
- (b) $h'(1)$, where $h(x) = f(\sqrt{x})$.

32. Estimate $\sqrt[3]{8.1}$ using linear approximation.

33. Let $f(x) = \frac{1}{1+x^2}$.

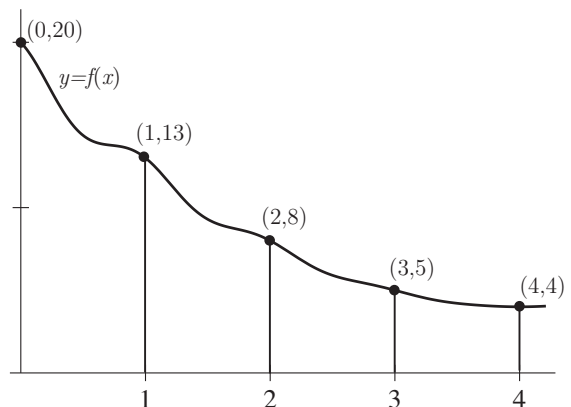
- (a) Find the local linearization of f near $x = 1$.
- (b) Use the local linearization you found to estimate $f(1.1)$.

34. Let C be the curve defined by the equation $xy + y^2 = 4$.

- (a) Verify that the point $(3, 1)$ lies on the curve C .
- (b) Find an equation for the tangent line to C at the point $(3, 1)$.
- (c) If the curve defines a function $f(x)$ near $x = 3$, then estimate $f(3.2)$.

35. Consider the following function f .

- (a) Find an approximation to the area under the graph of f , from $x = 0$ to $x = 4$, using a left endpoint Riemann sum with four rectangles.
- (b) Is your answer an overestimate of the actual area under the graph, or is it an underestimate? Please explain your reasoning.



36. Calculate $\int_1^5 [3f(x) + 4g(x)] dx$, given that

$$\int_1^2 f(x) dx = -1, \quad \int_2^5 f(x) dx = 1, \quad \int_{-1}^5 g(x) dx = -2, \quad \text{and} \quad \int_{-1}^1 g(x) dx = 3.$$

answer

$$\begin{aligned} \int_1^5 [3f(x) + 4g(x)] dx &= 3 \int_1^5 f(x) dx + 4 \int_1^5 g(x) dx \\ &= 3 \left(\int_1^2 f(x) dx + \int_2^5 f(x) dx \right) + 4 \left(\int_{-1}^5 g(x) dx - \int_{-1}^1 g(x) dx \right) \\ &= 3 \cdot (-1 + 1) + 4(-2 - 3) \\ &= -20. \end{aligned}$$

37. Consider the function $f(x) = x + \frac{1}{x}$ for $x \neq 0$.

- Find the critical points of f , and determine which are local minima of f , local maxima of f , or neither.
- Find the global maxima and minima of f .
- Over what intervals is f concave up? concave down?
- Where are the inflection points of f ?

38. Let $F(x) = \int_0^{3x} f(t) dt$, where $f(t) = 2^{(t^2)}$.

- Calculate $F(0)$.
- Using $n = 3$ subintervals and left-hand endpoints, estimate the value of $F(1)$. Begin by calculating Δt and filling in the table of values below.

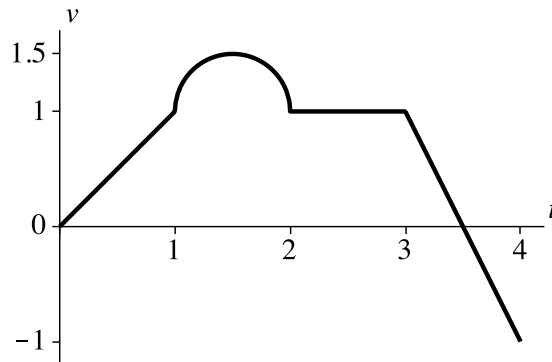
t				
$f(t)$				

$$\Delta t = \underline{\hspace{2cm}}$$

- Calculate $F'(x)$ and find the local linearization of $F(x)$ about $x = 0$.
- Using part (c), estimate the value $F(1)$ (this may not be a very good estimation).

39.

A particle's position at time t is given by $s(t)$, and its velocity at time t is given by $v(t)$. Given the graph of $v(t)$ below, if the particle has position $s = -1$ at $t = 1$, fill in the table of values for the position using *exact* values (i.e., without estimating). The graph below is made up of triangular, semicircular, and rectangular sections.



t	0	1	2	3	4
$s(t)$		-1			

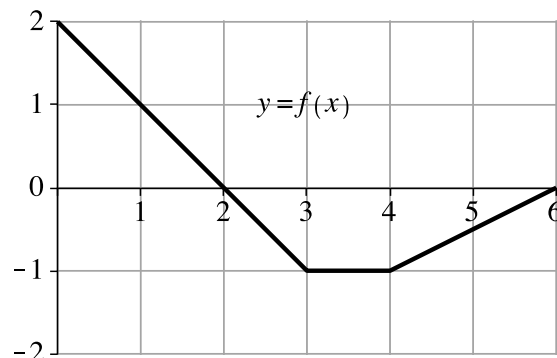
40. The volume of water remaining in a leaking tank is given by the function $V = 50e^{-0.1t}$, where V is measured in gallons and t in minutes.
- How fast is the tank leaking after 10 minutes? Include appropriate units.
 - How much water remains in the tank after 10 minutes? Include appropriate units.
41. The rate at which water is leaking from an initially full 100-gallon tank is given by the function $r = -7e^{-0.1t}$, where r is measured in gallons per minute and t is measured in minutes. (The negative sign indicates a decrease as opposed to an increase in volume.)
- How fast is the tank leaking after 10 minutes? Include appropriate units.
 - How much water remains in the tank after 10 minutes? Include appropriate units.
42. Let $F(x) = \int_2^x \frac{1}{\ln t} dt$. Find $F'(x)$. Is F increasing or decreasing? What can you say about the concavity of F ?
- 43.

Let $f(x)$ be the function whose graph is given to the right. Define

$$F(x) = \int_0^x f(t) dt.$$

Fill in the entries in the table that follows.

x	1	2	3	4	5
$F(x)$					
$F'(x)$					
$F''(x)$					



44. Let $f(x)$ be the function from the previous problem and $g(x)$ be given by the following table:

x	0	1	2
$g(x)$	2	-1	5
$g'(x)$	3	4	-2

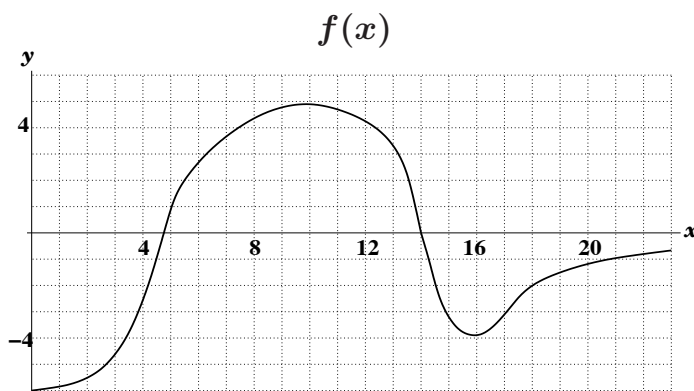
If $h(x) = g(f(x))$, what is $h'(2)$? If $k(x) = f(g(x))$, what is $k'(2)$?

45.

Let f be the function whose graph is given below, and define a new function F by the equation $F(x) = \int_0^x f(t) dt$.

Given below are several lists of numbers. Rank each list in order from smallest to largest.

- (a) $0, f'(3), f'(9), f'(14), f'(15)$
- (b) $0, F(3), F(4), F(6), F(13), F(14)$
- (c) $0, F'(0), F'(4), F'(7), F'(10), F'(15), F'(18), F'(23)$



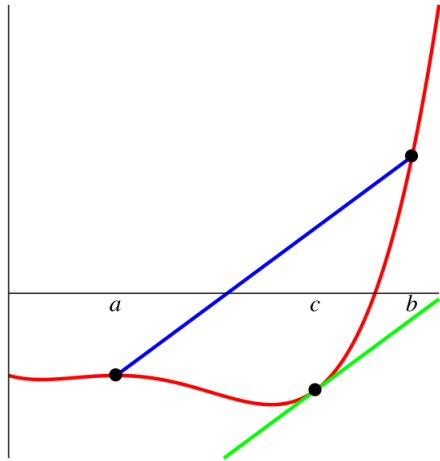
46. Find the slope of the tangent line to each of the following functions at $x = 2$.

- (a) $f(x) = \sin(\cos x)$

(b) $F(x) = \int_0^x e^{-t^2} dt$

(c) $f(x) = x^x$

47. State the Mean Value Theorem, and draw a picture to demonstrate the theorem visually.



48. Let $f(x) = (x - 1)^2 e^{-x}$.

(a) State the domain of $f(x)$.

(b) What is the behavior of $f(x)$ as $x \rightarrow +\infty$ and $x \rightarrow -\infty$?

(c) Find the intervals of increase and decrease of $f(x)$, and all local maxima and minima of $f(x)$.

(d) Find the inflection points, and the intervals where the function is concave up/down.

49. The acceleration due to gravity on the moon is $1.6 \frac{m}{s^2}$. An object is thrown upwards from a height of 2.4 m, with an initial velocity of $1.6 \frac{m}{s}$.

(a) What height does the object reach?

(b) How fast is the object traveling when it lands?

More practice from the last two chapters:

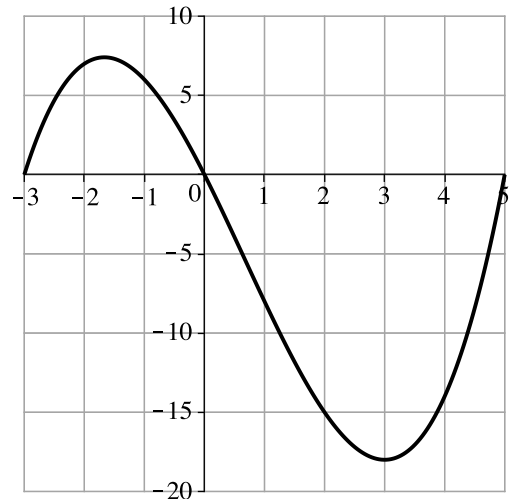
50. A village wishes to measure the quantity of water that is piped to a factory during a typical morning. A gauge on the water line gives the flow rate (in cubic meters per hour) at any instant. The flow rate is about $100 \text{ m}^3/\text{hr}$ at 6 am and increases steadily to about $280 \text{ m}^3/\text{hr}$ at 9 am.
- (a) Using only this information, give your best estimate of the total volume of water used by the factory between 6 am and 9 am.
- (b) How often should the flow rate gauge be read to obtain an estimate of this volume to within 6 m^3 ?

51.

Suppose a function $f(x)$ is graphed as below.

(a) Using just the figure, estimate $\int_{-3}^5 f(x) dx$.

(b) If you knew the function were given by the formula $f(x) = \frac{1}{2}x(x+3)(x-5)$, what would the integral be? How far off is your graphical estimate?



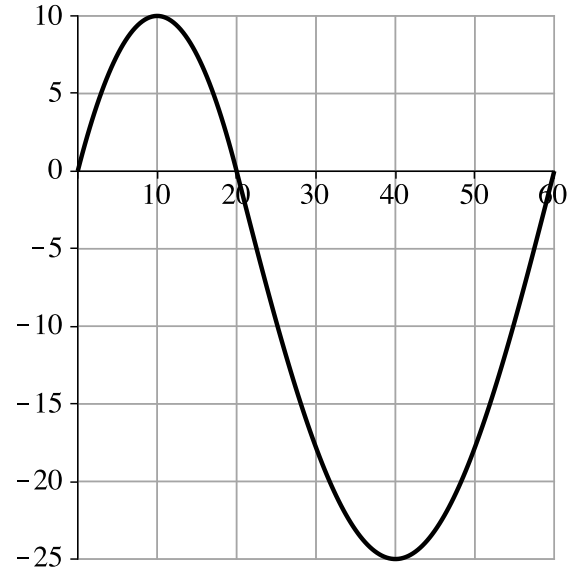
52. Find the area of the region between the line $y = 1$ and one arch of $y = \sin \theta$.
53. Find the area of the region between the parabola $y = 4 - x^2$ and the x -axis.
54. If $\int_2^5 (2f(x) + 3) dx = 17$, find $\int_2^5 f(x) dx$.
55. Find $\int_{-1}^1 |x| dx$ geometrically.
56. An old rowboat has sprung a leak. Water is flowing from the boat at a rate, $r(t)$, given in the following table.

t minutes	0	5	10	15
$r(t)$ liters/min	12	20	24	16

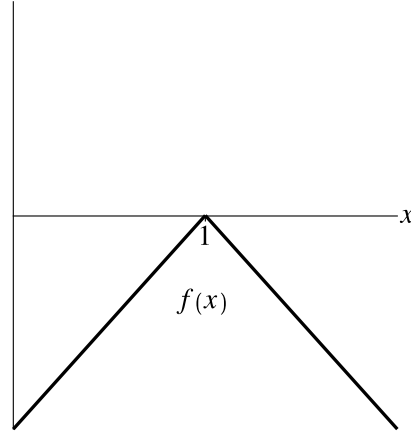
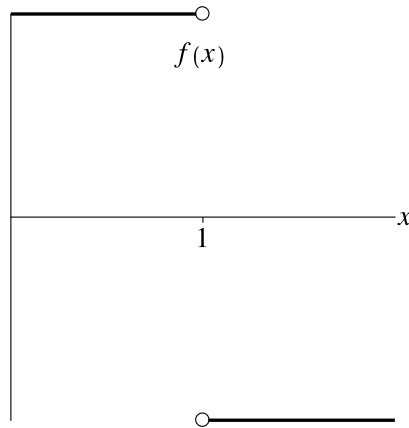
- (a) Compute upper and lower estimates for the volume of water that has flowed into the boat during the 15 minutes.
- (b) Draw a graph to illustrate the lower estimate.

57. A bicyclist is pedaling along a straight road for one hour with a velocity v shown in the figure. She starts out five kilometers from the lake and positive velocities take her toward the lake. Note that the horizontal units are in minutes while the vertical units are in kilometers per hour.

- (a) Does the cyclist ever turn around? If so, at what time(s)?
- (b) When is she going the fastest? How fast is she going then? Toward the lake or away?
- (c) When is she closest to the lake? Approximately how close to the lake does she get?
- (d) When is she farthest from the lake? Approximately how far from the lake is she then?

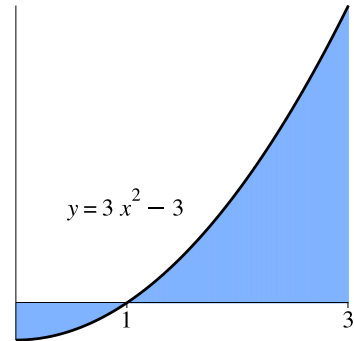


58. The area under $1/\sqrt{x}$ on the interval $1 \leq x \leq b$ is equal to 6. Find the value of b using the Fundamental Theorem.
59. Find the exact area of the region bounded by the x -axis and the graph of $y = x^3 - x$.
60. For each of the graphs $f(x)$ shown below, sketch a graph of $F(x)$ such that $F'(x) = f(x)$ and $F(0) = 1$.



61.

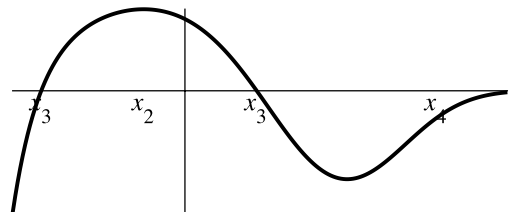
Find the area of the shaded region in the figure between $y = 3x^2 - 3$ and the x -axis.



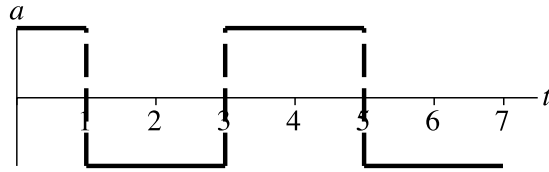
62. Sketch the parabola $y = x(x - \pi)$ and the curve $y = \sin x$, showing their points of intersection. Find the exact area between the two graphs.

63.

The graph of $f'(x)$ is given. Sketch a possible graph for $f(x)$. Mark the points x_1, \dots, x_4 on your graph and label local maxima, local minima, and inflection points on your graph.



64. The acceleration, a , of a particle as a function of time is shown in the figure. Sketch graphs of velocity and position against time. The particle starts at rest at the origin.



65.

The graphs of three functions are given in the figure. Determine which is f , which is f' , and which is $\int_0^x f(t) dt$. Explain your answer.

