1. The number of yeast cells in a laboratory culture increases rapidly initially but levels off eventually. The population is modeled by the function

$$n = f(t) = \frac{a}{1 + be^{-0.7t}}$$

where t is measured in hours. At time $t = 0$ the population is 20 cells and is increasing at a rate of 12 cells/hour. Find the values of a and b. According to this model, what happens to the yeast population in the long run?

2. Do problem 68 parts a)-e) on page 249-250 of the text.

3. Do problem 73 on page 250 of the text.

4. A boat at anchor is bobbing up and down in the sea. The vertical distance, y, in feet, between the sea floor and the boat is given as a function of time, t, in minutes by

$$y = 15 + 6\sin(2\pi t)$$

   (a) Find $\dfrac{dy}{dt}$.

   (b) Find $\dfrac{dy}{dt}$ when $t = \frac{5}{6}$. Explain in an English sentence what this means in terms of the movement of the boat. Include units.

5. If the position of a particle at time t is given by the formula $s(t) = t^3 - t$, what is the velocity of the particle at time $t = 1$?

6. A rock falling from the top of a vertical cliff drops a distance of $s(t) = 16t^2$ feet in t seconds. What is its speed at time t? What is its speed when it has fallen 64 feet?

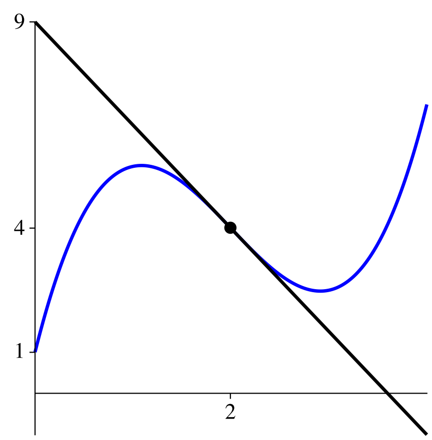
7. The height of a rock at time t which is thrown vertically from a height of 44 feet is given by the formula $s(t) = -t^2 + 20t + 44$. What is the maximum height of the rock? When does it hit the ground? What is the impact speed?

8. By increasing its advertising cost x(in thousands of dollars) for a product, a company discovers that it can increase the sales y(in thousands of dollars) according to the model

$$y = -\frac{1}{10}x^3 + 6x^2 + 400, \;\; 0 \le x \le 40.$$

Find the inflection point of this model and interpret in the context of the problem using complete English sentences.

9. The Figure shows the tangent line approximation to $f(x)$ near $x = a$.



(a) Find a, $f(a)$, $f'(a)$.

(b) Find an equation for $L(x)$, the tangent line approximation.

(c) Estimate $f(2.1)$ and $f(1.98)$ using linear approxiation (tangent line approximation). Are these under or overestimates? Which estimate would you expect to be more accurate and why?

10. Use linear approximation to estimate $\sqrt{24}$.

11. For $f(x) = x^3 - 3x^2$ on $-1 \leq x \leq 1$, find the critical points of f, the inflection points, the values of f at all these points and the endpoints, and the absolute maxima and minima of f. Then sketch the graph, indicating clearly where f is increasing or decreasing and its concavity.

12. For $f(x) = x + \sin x$ on $0 \leq x \leq 2\pi$, find the critical points of f, the inflection points, the values of f at all these points and the endpoints, and the absolute maxima and minima of f. Then sketch the graph, indicating clearly where f is increasing or decreasing and its concavity.

13. For $f(x) = \dfrac{4x^2}{x^2+1}$, find the critical points of f, the inflection points, the values of f at all these points, the limits as $x \to \pm\infty$, and the absolute maxima and minima of f. Then sketch the graph, indicating clearly where f is increasing or decreasing and its concavity.

14. Find the exact absolute maximum and minimum values of the function
$$h(z) = \frac{1}{z} + 4z^2 \text{ for } z > 0$$

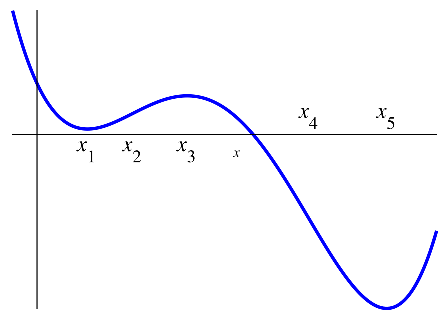
15. Use derivatives to identify local maxima and minima and points of inflection. Then graph the function.
$$f(x) = e^{-x^2}.$$

16. (a) Find all critical points and all inflection points of the function $f(x) = x^4 - 2ax^2 + b$. Assume a and b are positive constants.

(b) Find values of the parameters a and b if f has a critical point at the point $(2, 5)$.

(c) If there is a critical point at $(2, 5)$, where are the inflection points?

17. For the function, f, graphed in the Figure:



(a) Sketch $f'(x)$.

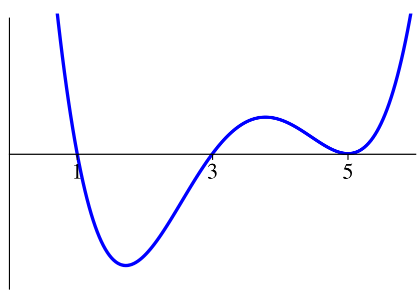
(b) Where does $f'(x)$ change its sign?

(c) Where does $f'(x)$ have local maxima or minima?

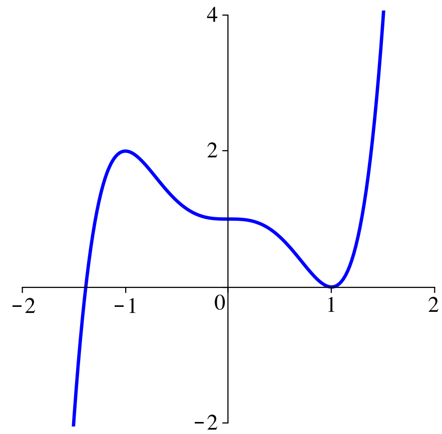
18. Using your answer to the previous problem as a guide, write a short paragraph (using complete sentences) which describes the relationships between the following features of a function f:

- The local maxima and minima of f.
- The points at which the graph of f changes concavity.
- The sign changes of f'.
- The local maxima and minima of f'.

19. The figure shows a graph of $y = f'(x)$ (not the function f). For what values of x does f have a local maximum? A local minimum?

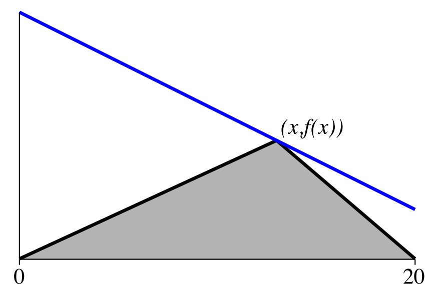


20. On the graph of f' shown in the figure, indicate the x-values that are critical points of the function f itself. Are they local minima, local maxima, or neither?



21. Find a formula for the function described: a cubic polynomial with a local maximum at $x = 1$, a local minimum at $x = 3$, a y-intercept of 5, and an x^3 term whose coefficient is 1.

22. Find the x-value maximizing the shaded area. One vertex is on the graph of $f(x) = x^2/3 - 50x + 1000$ (shown in blue).



23. Given that the surface area of a closed cylinder of radius r cm and height h cm is 8 cm^2, find the dimensions giving the maximum volume.

24. Do the last two problems from Project 11.

25. Find the range of the function $f(x) = x^3 - 6x^2 + 9x + 5$, if the domain is $0 \leq x \leq 5$.

26. A landscape architect plans to enclose a 3000 square foot rectangular region in a botanical garden. She will use shrubs costing \$45 per foot along three sides and fencing costing \$20 per foot along the fourth side. Find the minimum total cost.

27. A piece of wire of length L cm is cut into two pieces. One piece, of length x cm, is made into a circle; the rest is made into a square. Find the value of x that makes the sum of the areas of the circle and square a minimum. Find the value of x giving a maximum.

28. Water is being pumped into a vertical cylinder of radius 5 meters and height 20 meters at a rate of 3 meters^3/min. How fast is the water level rising when the cylinder is half full?

29. Does L'Hopital's rule apply to the limit? If so, evaluate it.

$$\lim_{x \to 0} \frac{x}{e^x}.$$

30. Does L'Hopital's rule apply to the limit? If so, evaluate it.

$$\lim_{x \to 1} \frac{\sin \pi x}{x - 1}.$$

31. Determine whether the limit exists, and if possible evaluate it.

$$\lim_{t\to 0} \frac{e^t - 1 - t}{t^2}$$

32. Determine whether the limit exists, and if possible evaluate it.

$$\lim_{t\to 0^+} \frac{3\sin t - \sin 3t}{3\tan t - \tan 3t}$$

33. True or false? If true, explain how you know. If false, give a counterexample. If $f'(p) = 0$, then $f(x)$ has a local minimum or local maximum at $x = p$.

34. True or false? If true, explain how you know. If false, give a counterexample. If $f''(p) = 0$, then the graph of f has an inflection point at $x = p$.

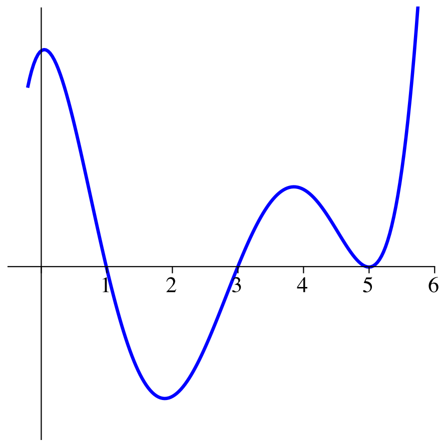
35. Sketch the graph of a function defined at every point of $[0, 2]$ which has an absolute minimum but no absolute maximum. Why does your graph not contradict the Extreme Value Theorem?

36. Do problem 68 parts a)-e) on page 249-250 of the text.

37. Do problem 73 on page 250 of the text.

38. Precisely state the Mean Value Theorem.

39. The graph of $y = f''(x)$ is shown. Where are the inflection points of $y = f(x)$?



40. Find the absolute maximum and minimum of the function $f(x) = 9x^{1/3} - x^3$ on the closed interval $[-1, 8]$.

41. Which point on the parabola $y = x^2 - x$ minimizes the square of the distance to the point $(1, 1)$?

42. Compute

$$\lim_{n\to\infty}\left(1+\frac{2}{n}\right)^n.$$

43. Compute

$$\lim_{x\to 1}\frac{\ln x}{x^2-1}.$$

44. Compute

$$\lim_{x\to 2}\frac{x^2-3x+2}{x+5}.$$

.

45. Let f be a continuous function with domain $[-1,3]$, and assume that f is differentiable on $(-1,3)$. Which of the following statements about f are definitely true?

(a) If f has a critical point at $x=2$, then f has either a local maximum or a local minimum at $x=2$.

(b) If f has a critical point at $x=1$ and $f''(x)<0$, then f has a local maximum at $x=1$.

(c) If f has a critical point at $x=1$ and $f''(x)<0$, then f has an absolute maximum at $x=1$.

(d) If $f(3)>f(-1)$, then f has an absolute maximum at $x=3$.

(e) If $f'(0)=0$ and $f(0)>f(3)>f(-1)$, then f has an absolute maximum at $x=0$.

(f) If the only critical point of f occurs at $x=2$, then f must have an absolute maximum at either $x=-1,2$, or 3.

(g) It is possible that f might not have an absolute minimum on this domain.

(h) If f has a local minimum at $x=2$ then $f'(2)=0$.

(i) If $f''(x)>0$ for all values of x in the domain, then $f(3)\geq f(-1)$.

(j) If $f''(x)>0$ for all values of x in the domain, then $f'(3)\geq f'(-1)$.

(k) If f has an inflection point at $x=1$ then $f'(x)$ has either a local maximum or a local minimum at $x=1$.

(l) If $f''(1)=0$ then f has an inflection point at $x=1$.

Derivative Practice.

Find $f'(x)$.

1. $f(x) = \dfrac{e^x}{\ln(x)}$

2. $f(x) = \ln(\cos(x))$

3. $f(x) = \arctan(\ln(\cos(x^2)))$

4. $f(x) = \arcsin(\sin(x))$

5. $f(x) = \arctan(4x + 3)$

6. $f(x) = (\arcsin(x))(\arctan(x))$

7. $f(x) = \arcsin(x \tan(x))$

8. $f(x) = e^{\arcsin(4x^2)}$

9. $f(x) = \ln(\arcsin(x)) + xe^{x^2}$

10. $f(x) = \tan^2(\arcsin(1))$

11. $f(x) = \sec(\ln(x))$

12. $f(x) = x^{\ln(x)}$