1. Name three different reasons that a function can fail to be differentiable at a point. Give an example for each reason, and explain why your examples are valid.

2. Given the following table of values, estimate $f'(0.6)$ and $f'(0.5)$; then use these to estimate $f''(0.6)$.

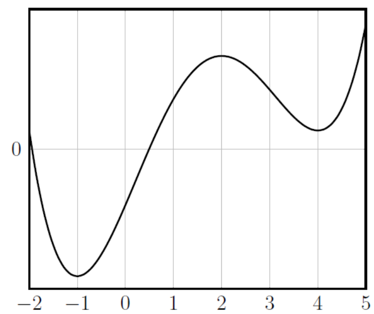
x	0	0.2	0.4	0.6	0.8	1.0
$f(x)$	3.7	3.5	3.5	3.9	4.0	3.9

3. If possible, draw the graph of a differentiable function with domain $[0, 6]$ satisfying $f'(x) > 0$ for $x < 1$, $f'(x) < 0$ for $x > 1$, $f'(x) > 0$ for $x > 4$, $f''(x) > 0$ for $x < 3$, and $f''(x) < 0$ for $x > 3$.

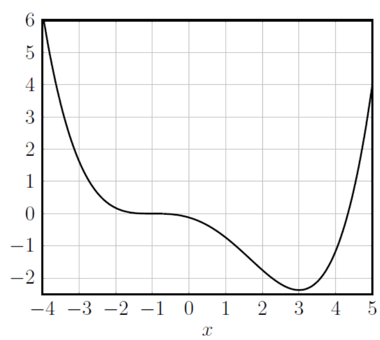
4. Given all of the following information about a function f sketch its graph.

   - $f(x) = 0$ at $x = -5, x = 0$, and $x = 5$
   - $\lim_{x\to-\infty} f(x) = \infty$
   - $\lim_{x\to\infty} f(x) = -3$
   - $f'(x) = 0$ at $x = -3, x = 2.5$, and $x = 7$

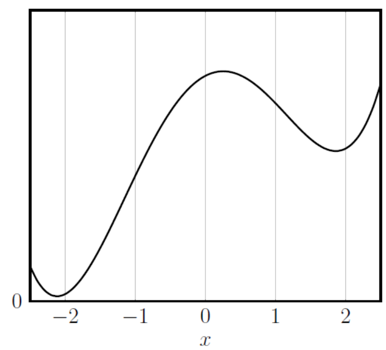
5. Given the graph of $y = f(x)$ shown, sketch the graph of the derivative.



6. If $y = f(x)$ is the graph shown below, sketch the graphs of $y = f'(x)$ and $y = f''(x)$. Estimate slopes as needed.



7. The graph of $y = f'(x)$ is shown. Where is $f(x)$ greatest? Least? Where is $f'(x)$ greatest? Least? Where is $f''(x)$ greatest? Least? Where is $f(x)$ concave up? Concave down?



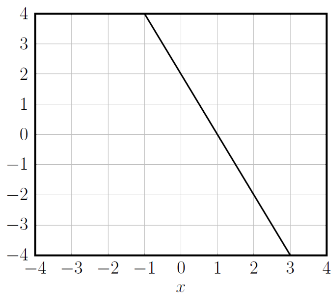
8. For what values of x is the graph of $y = x^5 - 5x$ both increasing and concave up?

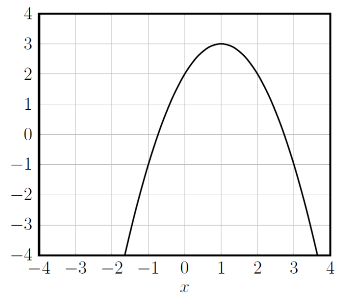
9. Where does the tangent line to $y = 2^x$ through $(0, 1)$ intersect the x-axis?

10. If $g(x) = e^x f(x)$, find and simplify $g''(x)$.

11. If $f(x) = 4x^3 + 6x^2 - 23x + 7$, find the intervals on which $f'(x) \geq 1$.

12. Let $f(x)$ and $g(x)$ be the functions graphed below:





Let $h(x) = f(x)g(x)$, let $j(x) = x^2 f(x)$, let $k(x) = f(x^2)$, let $p(x) = \frac{f(x)}{g(x)}$, let $q(x) = f(g(x))$, and $r(x) = g(g(x))$.

(a) Estimate $h'(1), h'(0), p'(0), q'(0), r'(1)$, and $r'(2)$.

(b) Estimate $j'(-1)$ and $k'(-1)$.

(c) Estimate all values of x for which $y = r(x)$ has a horizontal tangent line.

13. Using the information in the table, find:

x	1	2	3	4
$f(x)$	3	2	1	4
$f'(x)$	1	4	2	3
$g(x)$	2	1	4	3
$g'(x)$	4	2	3	1

(a) $h(4)$ if $h(x) = f(g(x))$

(b) $h'(4)$ if $h(x) = f(g(x))$

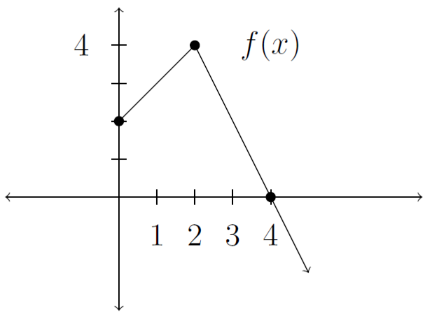
(c) $h(4)$ if $h(x) = g(f(x))$

(d) $h'(4)$ if $h(x) = g(f(x))$

(e) $h'(4)$ if $h(x) = \frac{g(x)}{f(x)}$

(f) $h'(4)$ if $h(x) = f(x)g(x)$

14. A graph of $f(x)$ is shown below. It is piecewise linear. The table below gives values of $g(x)$ and $g'(x)$.



x	0	1	2	3	4
$g(x)$	2	5	9	11	8
$g'(x)$	3	4	2	-3	-4

(a) Given $h(x) = f(x)g(x)$, find $h'(1)$.

(b) Given $k(x) = \frac{f(x)}{g(x)}$, find $k'(3)$.

(c) Given $l(x) = \frac{g(x)}{\sqrt{x}}$, find $l'(4)$.

(d) Given $m(x) = g(f(x))$, find $m'(3)$.

15. On what interval(s) is the function

$$f(x) = \frac{(5x+2)^3}{(2x+3)^3}$$

increasing?

16. If $g(2) = 3$ and $g'(2) = -4$, find $f'(2)$ for the following:

(a) $f(x) = x^2 - 4g(x)$

(b) $f(x) = \frac{x}{g(x)}$

(c) $f(x) = x^2 g(x)$

(d) $f(x) = g(x)^2$

(e) $f(x) = x \sin(g(x))$

(f) $f(x) = x^2 e^{g(x)}$

17. On what interval(s) is the function

$$f(x) = (x+3)e^{2x}$$

decreasing? On what intervals is it concave down?

18. Let $f(x) = xe^x$.

    (a) Find the interval(s) where f is increasing and the interval(s) where f is decreasing.

    (b) Find the interval(s) where f is concave up and the interval(s) where f is concave down.

19. Find the slope of the line tangent to $y = \cos(3\theta)$ at $\theta = -\dfrac{\pi}{18}$

20. Differentiate the following functions.

(a) $a(x) = x^4 + 2x^3 + 6x$

(b) $b(x) = \sqrt{x} + \dfrac{1}{3x^2}$

(c) $c(x) = (x^3 - x)e^x$

(d) $d(t) = \dfrac{t+2}{t+1}$

(e) $k(p) = \sec\left(e^{(x^2+2)}\right)$

(f) $d(t) = 2e^t + t^{2/5} - 2t^5 + t^{-3} - \pi^2$

(g) $h(x) = \dfrac{-4x^2 + 3x - 1}{4x^3 + 10}$

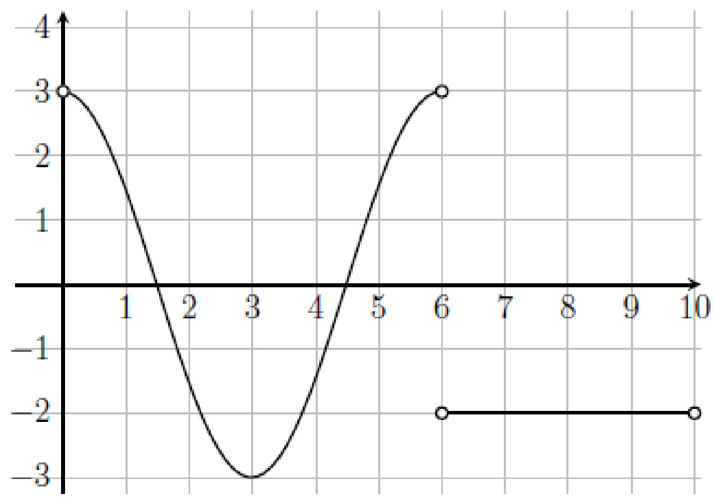
(h) $f(\theta) = \sec\left(\theta^2 + \cos(\theta)\right)$

(i) $n(w) = 4^w(w^2 + 11)$

21. Find the point on the graph of $f(x) = 3x^2 - 4x + 5$ such that the tangent line at that point is parallel to the line $y = 2x + 150$.

22. The graph below is the **derivative** of some function, f.

Graph of $f'(x)$



(a) On what intervals is f increasing?

(b) At what values of x does $f(x)$ have a local minimum?

(c) On what intervals is $f(x)$ concave up?

(d) At what values of x does $f(x)$ have an inflection point?

(e) Is it possible for the function $f(x)$ to be continuous at $x = 6$? Explain.

23. Let $f(x) = x^3 + \frac{9}{2}x^2 - 12x + 13$.

    (a) Find $f'(x)$ and $f''(x)$.

    (b) On what interval(s) is f decreasing?

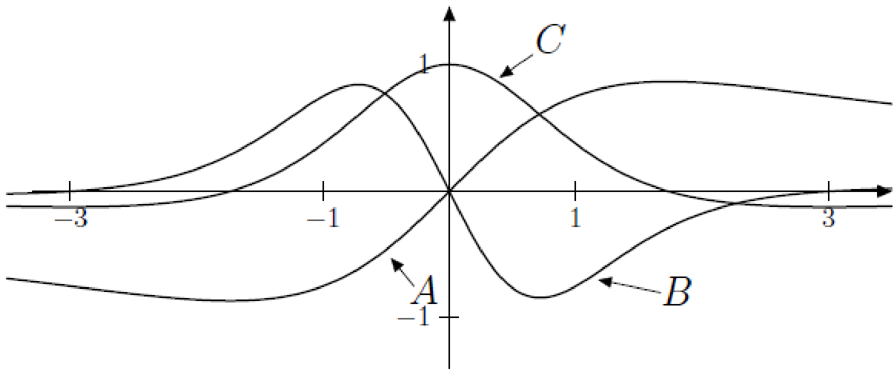
    (c) On what interval(s) is f concave downward?

24. Consider the curve described by the points satisfying the equation

$$x^3 + y^3 = 2x^3y + 5$$

Find the equation of the tangent line at the point (1,2).

25. The graph of a function $f(x)$ and its first and second derivatives are shown below.

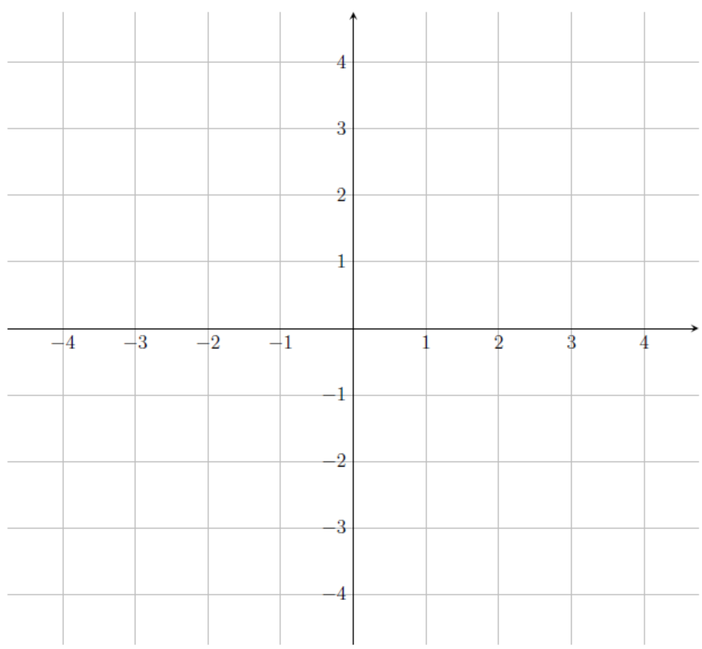


Explain which graph is $f(x), f'(x)$, and $f''(x)$.

26. Draw the graph of a function $f(x)$ that satisfies the following conditions:

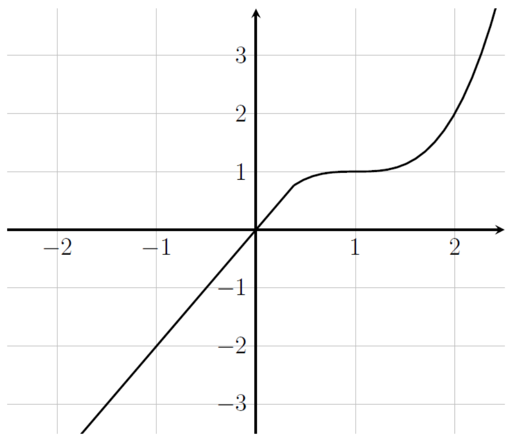
- $f(x) < 0$ when $x < -3$
- $f(x) > 0$ on the intervals $(-3, 0)$ and $(0, \infty)$
- $f(-3) = f(0) = 0$
- $f'(x) > 0$ when $x < -2$ and $x > 0$
- $f'(x) < 0$ on the interval $(-2, 0)$
- $f'(-2) = f'(0) = 0$
- $f''(x) > 0$ when $-1 < x < 2$
- $f''(x) < 0$ on the intervals $(-\infty, -1)$ and $(2, \infty)$
- $f''(-1) = f''(2) = 0$

Graph of $f(x)$



27. Find the derivatives of the following functions at the given point using the information given below:

Graph of $f(x)$



x	-3	-2	-1	0	1	2	3	4
$g(x)$	2	-4	-2.5	1	3	5	14	5
$g'(x)$	-3	-2	2	4	2	4	3	-6

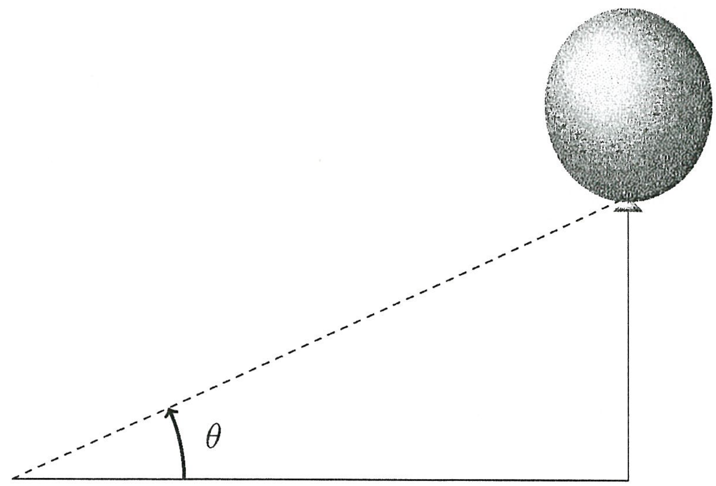
(a) If $h(x) = g(g(x))$, what is $h'(-3)$?

(b) If $q(x) = f(x)g(x)$, what is $q'(1)$?

(c) If $m(x) = g(\sqrt{x})$, what is $m'(4)$?

28. Hercules is attempting to scale a Roman wall, so he leans a 13 meter ladder against the vertical wall. While he is climbing the ladder, the goddess Juno kicks the bottom of the ladder away from the wall causing the top of the ladder to start sliding down the wall. When the top of the ladder is 5 meters above the ground, the top of the ladder is sliding down the wall at a rate of 2 meters per second. At this same point in time, what is the speed of the bottom of the ladder as it moves away from the wall? Be sure to include units in your answer.

29. An observer stands 200 m from the launch site of a hot-air balloon. The balloon rises vertically at a constant rate of 4 m/s. How fast is the angle of elevation of the balloon increasing 50 s after launch?



30. A street light is mounted at the top of a 15 ft tall pole. A boy 5 ft tall walks away from the pole at a speed of 3 ft/sec along a straight path. How fast is the tip of his shadow moving when he is 40 ft from the pole?

31. Given the curve $e^y = 2x^3y^2 + e^2$, find $\dfrac{dy}{dx}$ in terms of x and y.

32. If $f(x) = x^2 + 1$ and $g(x) = 5 - x$, find:

    (a) $h'(1)$ if $h(x) = f(x) \cdot g(x)$

    (b) $j'(2)$ if $j(x) = \frac{f(x)}{g(x)}$

    (c) $k'(3)$ if $k(x) = f(g(x))$

33. Find the equations for the lines tangent to the graph of $xy + y^2 = 4$ when $x = 3$.

34. Find $\dfrac{dy}{dx}$ if $x^3 + y^3 - 4x^2y = 0$.

35. Consider the curve $x^2 + 2xy + 5y^2 = 4$. At what point(s) is the tangent line to this curve horizontal? At what point(s) is the tangent line to this curve vertical? At what points is the slope of the tangent line equal to 2?

36. Consider the function $y + \sin(y) + x^2 = 9$. Find the slope of the curve at $(3, 0)$.

37. A radio navigation system used by aircraft gives a cockpit readout of the distance, s, in miles, between a fixed ground station and the aircraft. The system also gives a readout of the instantaneous rate of change, $\frac{ds}{dt}$, of this distance in miles/hour. An aircraft on a straight flight path at a constant altitude of 10,560 feet (2 miles) has passed directly over the ground station and is now flying away from it. What is the speed of this aircraft along its constant altitude flight path when the cockpit readouts are s=4.6 miles and $\frac{ds}{dt}$=210 miles/hour?

38. A gas station stands at the intersection of a north-south road and an east-west road. A police car is traveling west toward the gas station. The speed of the police car is 100 mph at the moment it is 3 miles from the gas station. At the same time, the truck is 4 miles north from the gas station going 80 mph. At this moment, is the distance between the car and truck increasing or decreasing? How fast? (Distance is measured along a straight line joining the car and truck.)

39. Is
$$f(x) = \begin{cases} 2x+1 & ,\text{when } x \geq 3 \\ x^2+2 & ,\text{when } x < 3 \end{cases}$$
differentiable at $x = 3$?

40. Is
$$f(x) = \begin{cases} 2x+1 & ,\text{when } x < 0 \\ (x+1)^2 & ,\text{when } x \geq 0 \end{cases}$$
differentiable at $x = 0$?

41. Is
$$f(x) = \begin{cases} x+1 & ,\text{when } x \leq 0 \\ 1-x^2 & ,\text{when } x > 0 \end{cases}$$
differentiable at $x = 0$?

42. Is $f(x) = x^{\frac{1}{3}}$ differentiable at $x = 0$?

Derivative Practice

Find the derivative of each function.

1. $f(x) = \dfrac{x^2+1}{5}$

2. $f(x) = \pi^3$

3. $r(t) = \sqrt{t} + \frac{1}{3t}$

4. $c(q) = \dfrac{1+q^2+q^3+q^4+q^5+q^6}{q^3}$

5. $k(x) = \dfrac{1}{x^2}$

6. $g(p) = (p+1)(2p-1)$

7. $f(x) = xe^x$

8. $y(x) = \sin(x)\cos(x)$

9. $f(x) = 13(2^{x-3}) + x^{2/3} - x^{-3/2}$

10. $m(x) = kx^n$

11. $f(x) = \dfrac{2x-1}{x+3}$

12. $c(t) = t^2\cos(t) + 4\sin(t)$

13. $s(t) = \dfrac{\sin(t)\cos(t)}{1 + t\tan(t)}$

14. $h(x) = \cos^2(x) + \sin^2(x)$

15. $P(t) = 2\sin^2(t)$

16. $f(x) = \dfrac{\sec(x)}{1 + \tan(x)}$

17. $s(t) = \dfrac{t^2 + 1}{t + 1}$

18. $R(q) = (2q^7 - q^2) \cdot \dfrac{q - 1}{q + 1}$

19. $P(t) = 432e^{-0.043t}$

20. $f(x) = 2e^{\cos(3x^2)}$

21. $y = \dfrac{e^{10x^2}\sin(20x)}{\sqrt{x^2 + 3\cos(x) - 1}}$