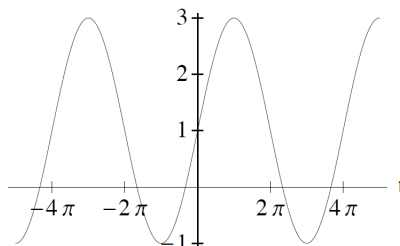


1. Consider the trigonometric function  $f(t)$  whose graph is shown below. Write down a possible formula for  $f(t)$ .



2. Find the domain of the function  $f(x) = \sqrt{4 - 5x}$ .
3. What is the domain and range of the function  $f(x) = (x + 3)^2 - 4$ ?
4. Use the limit definition of derivative to compute the derivative of the function  $f(x) = \frac{4}{x}$  at  $x = 2$ .
5. The table below gives the depth of snow that has fallen in inches as a function of time in hours past 8am. What is  $d(1.5)$  (including units) and what does it represent? What time did it start snowing? Next, estimate  $d'(1)$  and  $d'(2)$ . Include units in your answer and say in a full English sentence what the meaning of these numbers are.

$x$	.75	1	1.25	1.5	1.75	2	2.25	2.5
$d(x)$	0	.3	.8	1.25	1.85	2.0	2.2	2.4

6. Say  $f(x) = 3x^2 + x$ . Use the definition of derivative to find  $f'(1)$ . Then write the equation of the tangent line to the curve at  $x = 1$ .
7. Say that the position of an object moving horizontally is given by  $s(t) = \sqrt{6t + 7}$ , where position is measured in miles and time is measured in hours. Find the instantaneous velocity at an arbitrary  $t = a$ , and then the instantaneous velocity at  $t = 7$ . Include units.

8. Consider the function

$$f(x) = \begin{cases} x^2 + \cos x & \text{when } -5 < x < 0 \\ 5 & \text{when } x = 0 \\ e^{3x/2} & \text{when } 0 < x < 5 \end{cases}$$

- (a) Define what it means for a function  $f(x)$  to be continuous at a point  $x = a$ .
- (b)  $\lim_{x \rightarrow 2^+} f(x) =$
- (c)  $\lim_{x \rightarrow 0^+} f(x) =$
- (d)  $\lim_{x \rightarrow 0^-} f(x) =$
- (e)  $\lim_{x \rightarrow 0} f(x) =$
- (f) Is  $f(x)$  continuous at  $x = 0$ ? Fully explain your answer. If it is not continuous, state what type of discontinuity it is and why.

9. Sketch the graph of a function  $g(x)$  that satisfies ALL of the following properties:

- (a)  $\lim_{x \rightarrow 1^+} g(x) = 2$
- (b)  $\lim_{x \rightarrow 1} g(x)$  does not exist.
- (c)  $\lim_{x \rightarrow \infty} g(x) = -4$
- (d)  $\lim_{x \rightarrow -4^+} g(x) = -\infty$
- (e)  $\lim_{x \rightarrow -\infty} g(x) = \infty$

10. A car is first driven at an increasing speed and then the speed begins to decrease until the car stops. Sketch a graph of the distance the car has traveled as a function of time.
11. Find the average velocity over the interval  $0.2 \leq t \leq 0.3$  of a car whose position  $s(t)$  is given by the following table. Then estimate the velocity at  $t = 0.3$ . Include units.

$t$ (sec)	0.1	0.2	0.3	0.4	0.5
$s(t)$ (ft)	0	0.5	1.0	1.8	2.8

12. The function  $f(t) = -16t^2 + 64t$  gives the distance above the ground of a ball that is thrown from ground level straight up into the air at time  $t = 0$ , with an initial velocity of 64 ft/sec.
- (a) How high is the ball above the ground at  $t = 1$  second?
- (b) How fast is the ball moving at  $t = 1$ ,  $t = 2$ ,  $t = 3$  and  $t = 4$  seconds?
- (c) What is the maximum height of the ball? (Hint: you should have gotten a velocity of 0 for one of the values of  $t$  in the last part)
13. Precisely state the Intermediate Value Theorem. A polynomial  $p(x)$  has  $p(-2) = 13$  and  $p(0) = -1$  and  $p(1) = 1$ . Show that  $p(x)$  has at least two zeroes.

14. Find the following limits. Show all of your work.

(a)  $\lim_{x \rightarrow 2} \frac{x^2 + 4x - 12}{x - 2}$

(b)  $\lim_{x \rightarrow 9} \frac{3 - \sqrt{x}}{x - 9}$

(c)  $\lim_{x \rightarrow 4} \frac{\frac{2}{x} - \frac{1}{2}}{x - 4}$

(d)  $\lim_{x \rightarrow \infty} \frac{3x^4 + 2x - 1}{4 - x^4}$

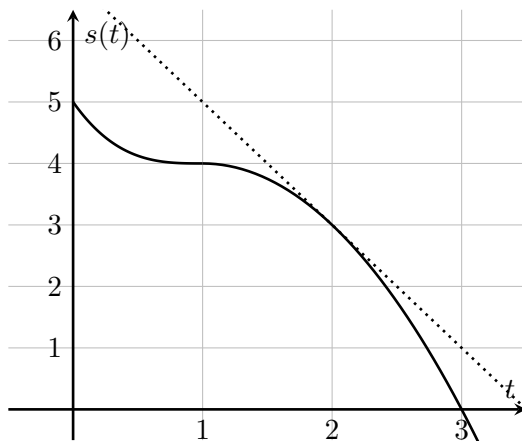
(e)  $\lim_{x \rightarrow \infty} \frac{3x^2 + 2x - 1}{4 - x}$

(f)  $\lim_{x \rightarrow \infty} \frac{\sqrt{4x^2 + 2x - 1}}{x + 1}$

(g)  $\lim_{x \rightarrow -\infty} (\sqrt{9x^2 + x} - 3x)$

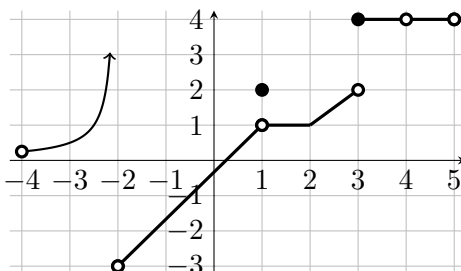
(h)  $\lim_{x \rightarrow \infty} (\sqrt{9x^2 + x} - 3x)$

15. The curve below shows position  $s$ , measured in feet, as a function of time  $t$ , measured in seconds. The dotted line is tangent to the curve.



- (a) Find the average velocity over the interval from  $t = 0$  to  $t = 3$ . Include units.  
(b) Find the instantaneous velocity at  $t = 2$ . Include units.  
(c) Find the equation of the tangent line.

16. Here's a graph of the function  $f(x)$ :



Meanwhile, here is a definition of the function  $g(x)$ :

$$g(x) = \begin{cases} -(x-3)^2 + 1 & x > 3 \\ -x + 6 & x < 3 \end{cases}$$

- Explain why the graph defines  $f$  as a function of  $x$ .
  - Is the  $f$  function invertible?
  - Is the function  $g$  invertible?
  - What is the domain and the range of  $f$ ?
  - What is the domain and the range of  $g$ ?
  - Find the limit of  $f(x)$  as  $x$  approaches  $-2$ , as  $x$  approaches  $1$ , as  $x$  approaches  $2$ , as  $x$  approaches  $3$ , as  $x$  approaches  $4$ . Clearly explain your results.
  - What is the limit of  $g(x)$  as  $x$  approaches  $3$ ? What about as  $x$  approaches  $0$ ?
  - List all of the discontinuities of  $f$  and  $g$  and clearly explain why these are discontinuities. State what type of discontinuities they are and why.
  - $\lim_{x \rightarrow 2} (f(x) + g(x)) =$
  - $\lim_{x \rightarrow 3} (f(x) + g(x)) =$
17. Find two values of the constant  $b$  so that the following function is continuous:
- $$h(x) = \begin{cases} x^2 + x - 11 & x \geq b \\ -3x + 1 & x < b \end{cases}$$
18. If  $4x - 9 \leq f(x) \leq x^2 - 4x + 7$  for  $x \geq 0$ , then find  $\lim_{x \rightarrow 4} f(x)$ .
19. Suppose  $f(2) = 3$  and  $f'(2) = 1$ . Find  $f(-2)$  and  $f'(-2)$  if  $f$  is assumed to be even.
20. Given all of the following information about a function  $f$ , sketch its graph.
- $f(x) = 0$  at  $x = -5$ ,  $x = 0$ , and  $x = 5$
  - $\lim_{x \rightarrow -\infty} f(x) = \infty$
  - $\lim_{x \rightarrow \infty} f(x) = -3$
  - $f'(x) = 0$  at  $x = -3$ ,  $x = 2.5$ , and  $x = 7$