Math 1300, Midterm 2

October 10, 2016

PRINT YOUR NAME: **SOLUTIONS**

PRINT INSTRUCTOR'S NAME: _____

Mark your section/instructor:

☐	Section 001	Brendt Gerics	8:00-8:50
☐	Section 002	Sebastian Bozlee	8:00-8:50
☐	Section 003	Albert Bronstein	10:00-10:50
☐	Section 004	Albert Bronstein	9:00-9:50
☐	Section 005	Suzanne Craig	9:00-9:50
☐	Section 006	Athena Sparks	10:00 - 10:50
☐	Section 007	Isabel Corona	10:00 - 10:50
☐	Section 008	Ali Lotfi	11:00 - 11:50
☐	Section 009	Krisztina Dearborn	11:00 - 11:50
☐	Section 010	Braden Balentine	11:00 - 11:50
☐	Section 011	Harrison Stalvey	12:00 - 12:50
☐	Section 012	Jun Hong	12:00 - 12:50
☐	Section 013	Tyler Schrock	1:00 - 1:50
☐	Section 014	Ira Becker	1:00 - 1:50
☐	Section 015	Carlos Pinilla-Suarez	2:00 - 2:50
☐	Section 016	Shen Lu	2:00 - 2:50
☐	Section 017	Matthew Pierson	3:00 - 3:50
☐	Section 018	Tien Trinh	9:00 - 9:50
☐	Section 019	Daniel Martin	4:00 - 4:50
☐	Section 880	Ira Becker	12:00 - 12:50
☐	Section 888R	Ilia Mishev	2:00 - 2:50
☐	Section 430R	Patrick Newberry	10:00 - 10:50

Run LaTeX again to produce the table

- No calculators or cell phones or other electronic devices allowed at any time.
- Show all your reasoning and work for full credit, except where otherwise indicated. Use full mathematical or English sentences.
- You have 80 minutes and the exam is **??** points.
- You do not need to simplify numerical expressions. For example leave fractions like 100/7 or expressions like ln(3)/2 as is.
- When done, give your exam to your instructor, who will mark your name off on a photo roster.

- We hope you show us your best work!

1. Differentiate the following functions. **Do not simplify.**

   (a) (4 points) $f(x) = 2e^x + x^{\frac{2}{5}} - 2x^5 + x^{-3} - \pi^2$

   Ans: $f'(x) = 2e^x + \frac{2}{5}x^{\frac{-3}{5}} - 10x^4 - 3x^{-4}$

   (b) (4 points) $h(x) = \dfrac{-4x^2 + 3x - 1}{4x^3 + 10}$

   Ans: $h'(x) = \dfrac{(4x^3 + 10)(-8x + 3) - (-4x^2 + 3x - 1)(12x^3)}{(4x^3 + 10)^2}$

   (c) (4 points) $g(x) = \ln(x) \cdot \arcsin(x)$

   Ans: $g'(x) = \dfrac{\arcsin(x)}{x} + \dfrac{\ln(x)}{\sqrt{1 - x^2}}$

2. **Multiple choice**. You do not need to show your work. Circle the derivative of the given function.

(a) (4 points) $f(\theta) = \sec(\theta^2 + \cos(\theta))$

A. $\sec(\theta)\tan(\theta)\,(2\theta - \sin(\theta))$

B. $\tan^2(2\theta - \sin(\theta))$

C. $\dfrac{2\theta - \sin(\theta)}{\cos(\theta^2 + \cos(\theta))}$

D. $\sec(\theta^2 + \cos(\theta))\tan(\theta^2 + \cos(\theta))(2\theta - \sin(\theta))$

Ans: D

(b) (4 points) $g(t) = \dfrac{\log_3(t)}{5}$

A. $\dfrac{t}{\ln(3)5}$

B. $\dfrac{\ln(3)}{5t}$

C. $\dfrac{1}{\ln(3)5t}$

D. $\dfrac{t}{\ln(3)}$

Ans: C

(c) (4 points) $h(w) = 4^w(w^2 + 11)$

A. $\ln(4)4^w(2w)$

B. $\ln(4)4^w(w^2 + 11) + 4^w(2w)$

C. $w4^{w-1}(w^2 + 11) + 4^w(2w)$

D. $\ln(4)4^w + 2w$

Ans: B

3. (8 points) Find the point on the graph of $f(x) = 3x^2 - 4x + 5$ such that the tangent line at that point is parallel to the line $y = 2x + 150$.

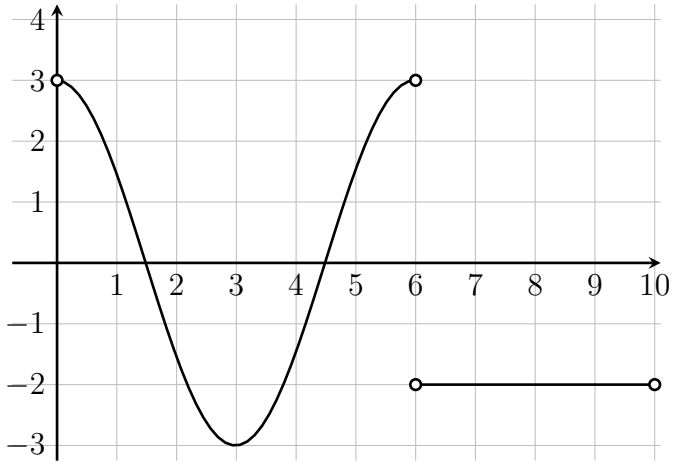
   Ans: $(1, 4)$

4. (8 points) Suppose $q(x) = x \arctan(x)$. Find the equation of the tangent line through $(1, \pi/4)$.

   Ans: $\left(y - \frac{\pi}{4}\right) = \left(\frac{\pi}{4} + \frac{1}{2}\right)(x - 1)$

5. **Multiple Choice** – circle the correct answer. The graph below is the **derivative** of some function, f.

Graph of $f'(x)$



(a) (2 points) On which one of the following intervals is f increasing?

A. $(0, 1.5)$ B. $(1.5, 3)$ C. $(1.5, 4.5)$ D. $(3, 4.5)$ E. $(6, 10)$

Ans: A

(b) (2 points) At which of the following x-values does $f(x)$ have a local minimum?

A. 1.5 B. 3 C. 4.5 D. 6 E. 8

Ans: C

(c) (2 points) On which one of the following intervals is $f(x)$ concave upward?

A. $(0, 1.5)$ B. $(1.5, 4.5)$ C. $(3, 6)$ D.$(6, 10)$ E. None.

Ans: C

(d) (2 points) Which of the following is an x-value of an inflection point?

A. 1.5 B. 3 C. 4.5 D. 6 E. 8

Ans: B

(e) (2 points) Is it possible for the function $f(x)$ to be continuous at $x = 6$? Explain.

Ans: Yes. At $x = 6$ the function $f(x)$ is not differentiable, but this does not imply it is not continuous. For example, there could be a corner or a cusp at $x = 6$.

6. Let $f(x) = x^3 + \frac{9}{2}x^2 - 12x + 13$. You must show your work for each of the following to receive credit.

   (a) (4 points) Find $f'(x)$ and $f''(x)$.

   Ans: $f'(x) = 3x^2 + 9x - 12$ and $f''(x) = 6x + 9$

   (b) (4 points) On what interval(s) is f decreasing?

   Ans: $(-4, 1)$

   (c) (4 points) On what interval(s) is f concave downward?

   Ans: $(-\infty, -3/2)$

7. (8 points) Consider the curve described by the points satisfying the equation

$$x^3 + y^3 = 2x^3y + 5.$$

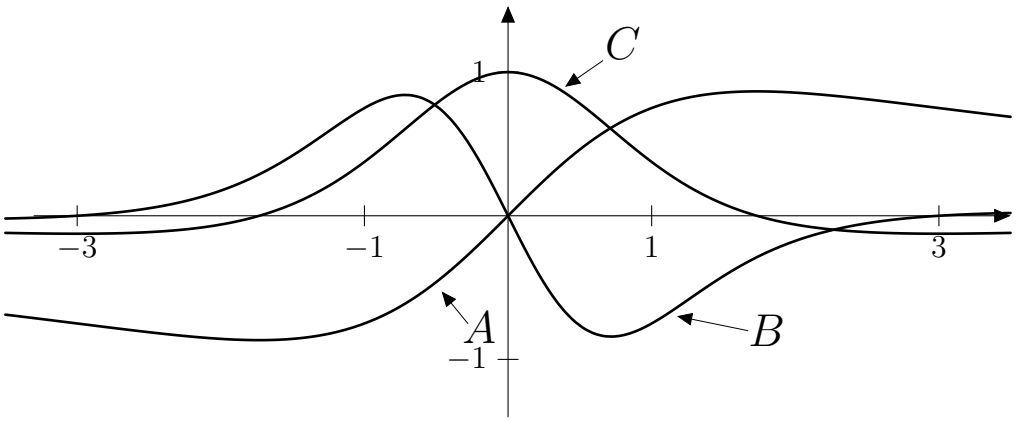
Find the equation of the tangent line at the point $(1, 2)$.

Ans: $(y - 2) = \dfrac{9}{10}(x - 1)$

8. (8 points) Use logarithmic differentiation to find y', where $y = x^{\sin(x)}$. Your final answer should be in terms of x.

Ans: $y' = \left(\cos(x)\ln(x) + \dfrac{\sin(x)}{x}\right) x^{\sin(x)}$

9. (6 points) The graph of a function $f(x)$ and its first and second derivatives are shown below.



Match the graph of each function A, B, and C to the corresponding functions below.

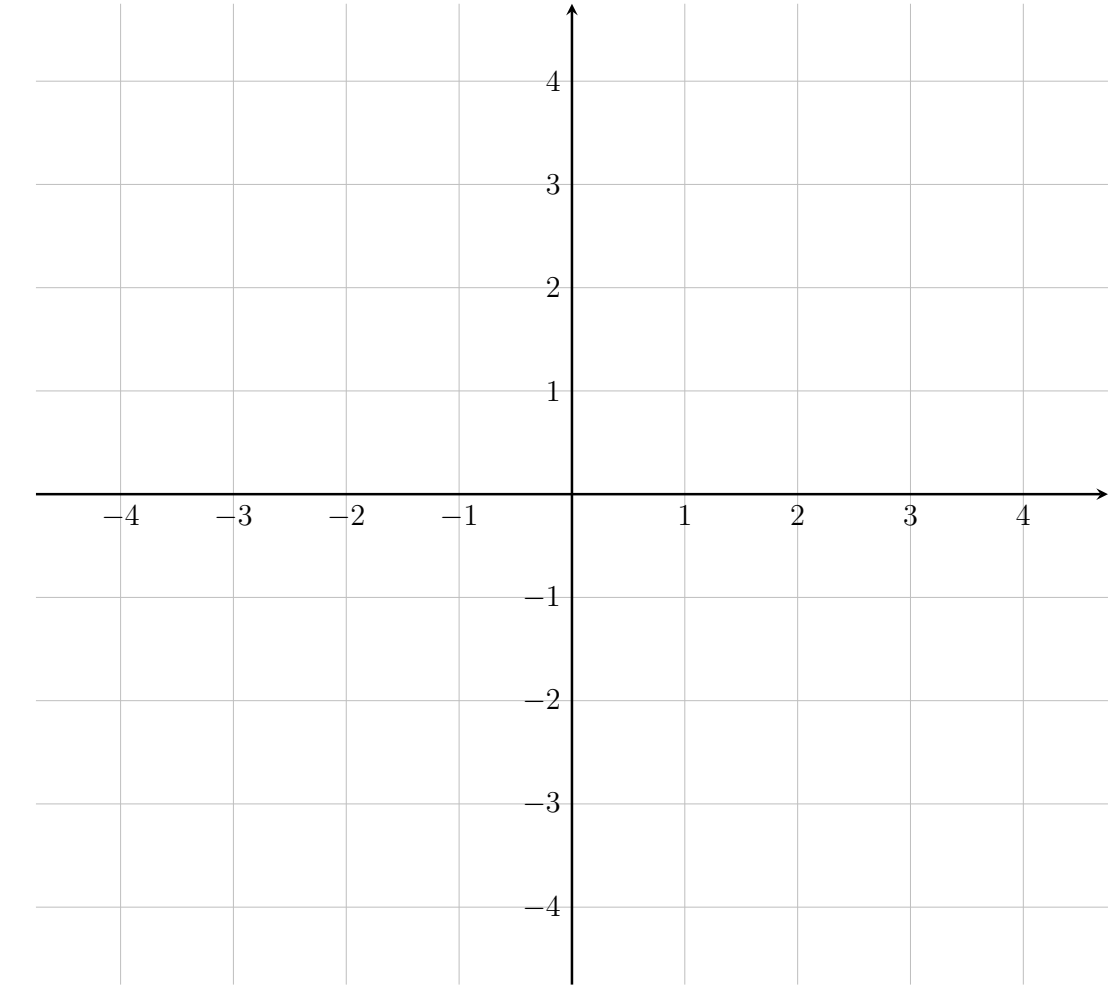
I) $f(x)$ _____ Ans: A

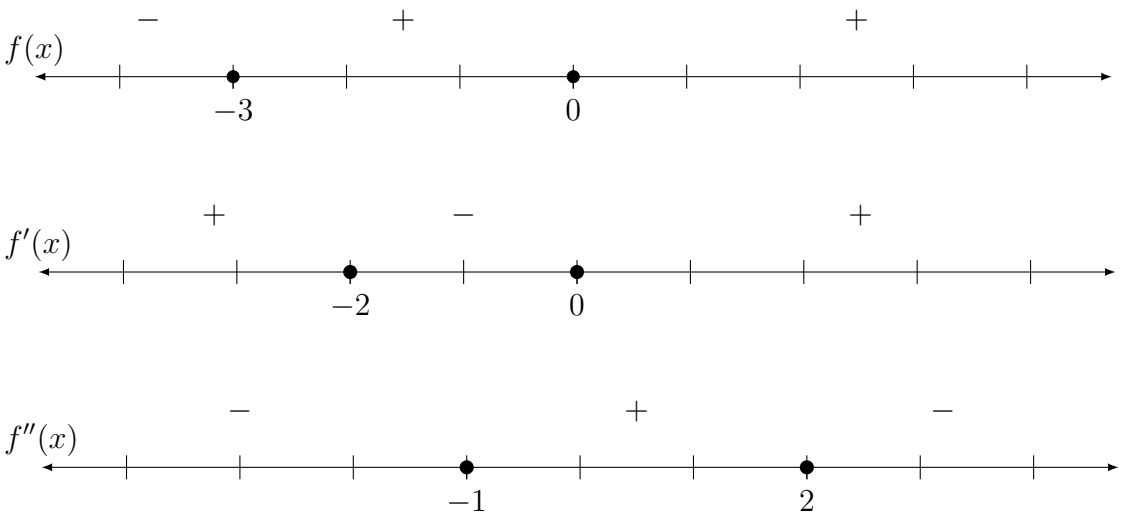
II) $f'(x)$_____ Ans: C

III) $f''(x)$_____ Ans: B

10. (8 points) Draw the graph of a function $f(x)$ that satisfies the given sign charts below. A solid dot indicates that the given function is zero at that value.

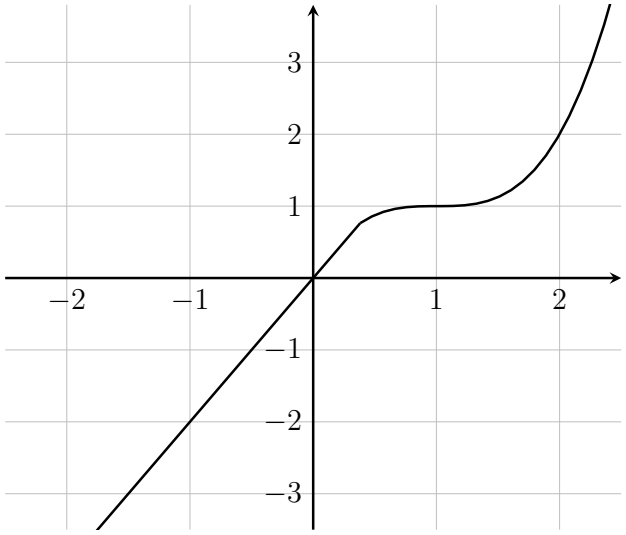
Graph of $f(x)$





11. **Mutliple choice** – circle the correct answer. Find the derivatives of the following functions at the given point using the information given below:

Graph of $f(x)$



x	-3	-2	-1	0	1	2	3	4
$g(x)$	2	-4	-2.5	1	3	5	14	5
$g'(x)$	-3	-2	2	4	2	4	3	-6

(a) (2 points) If $h(x) = g(g(x))$, what is $h'(-3)$?

A. -12 B. -3 C. 4 D. 5 E. 12

Ans: A

(b) (2 points) If $q(x) = f(x)g(x)$, what is $q'(1)$?

A. 0 B. 1 C. 2 D. 3 E. 4

Ans: C

(c) (2 points) If $p(x) = f^{-1}(x)$, what is $p'(-2)$?

A. -1 B. 2 C. $\frac{1}{2}$ D. -2 E. 0

Ans: C

(d) (2 points) If $m(x) = g(\sqrt{x})$, what is $m'(4)$?

A. $\frac{5}{4}$ B. 4 C. 5 D. -6 E. 1

Ans: E