

1. For each of the following statements, determine if they are true or false. If true, show or explain why it is true. If false, explain why or give a counterexample.

(a) If $\lim_{x \rightarrow a} g(x) = 0$ then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ does not exist.

(b) If $\lim_{x \rightarrow a} g(x) = \infty$ and $\lim_{x \rightarrow a} f(x)$ exists then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = 0$.

(c) If $\lim_{x \rightarrow a} f(x)$ exists and $\lim_{x \rightarrow a} (f(x) + g(x))$ exists then $\lim_{x \rightarrow a} g(x)$ exists.

(d) If f is a function with $f(a) > 0$ and $f(b) < 0$, then $f(x) = 0$ for some $a < x < b$.

(e) If $f(x) \leq g(x) \leq h(x)$ for all x with $\lim_{x \rightarrow a} f(x) = -1$ and $\lim_{x \rightarrow a} h(x) = 1$, then $\lim_{x \rightarrow a} g(x)$ exists with $-1 \leq \lim_{x \rightarrow a} g(x) \leq 1$.