Math 1300, Midterm 1

June 13, 2016

PRINT your name: _____

PRINT instructor's name: _____

Mark your section/instructor:

☐	Section 400	Jun Hong	9:15 - 10:35
☐	Section 401	Brendt Gerics	9:15 - 10:35
☐	Section 402	Braden Balentine	11:00-12:20
☐	Section 403	Matthew Pierson	12:45-2:05
☐	Section 005	Albert Bronstein	12:45-2:15 - 10:50

Question	Points	Score
1	12	
2	13	
3	12	
4	12	
5	12	
6	13	
7	13	
8	13	
Total:	100	

- No calculators or cell phones or other electronic devices allowed at any time.

- Show all your reasoning and work for full credit, except where otherwise indicated. Use full mathematical or English sentences.

- You have 90 minutes and the exam is 100 points.

- You do not need to simplify numerical expressions. For example leave fractions like 100/7 or expressions like ln(3)/2 as is.

- When done, give your exam to your instructor, who will mark your name off on a photo roster.

- We hope you show us your best work!

1. A woman goes for a morning run. The function $s(t)$ gives the distance in feet she has traveled after t seconds.

t in seconds	0	1	2	3	4	5	6
$s(t)$ in feet	0	7	18	31	42	54	67

  (a) (8 points) Compute the average velocity of the woman over the following time intervals. Be sure to include units.

    i $[2, 3]$

    ii $[3, 4]$

  (b) (4 points) Estimate the instantaneous velocity at $t = 3$. Be sure to include units.

2. (13 points) Find the value of c such that

$$f(x) = \begin{cases} -3x^2 + 2x + 4 & : \quad x \leq 1 \\ cx^2 - 5x + 7 & : \quad x > 1 \end{cases}$$

is continuous at $x = 1$. Please fully justify your answer using the definition of continuity.

3. Multiple Choice: Evaluate the following limits. Circle the correct answer. You do **not** need to show work.

(a) (3 points) $\lim\limits_{x \to \pi} \dfrac{x - 2\pi}{x + \pi}$

I) $\dfrac{1}{2}$ II) $-\dfrac{1}{2}$ III) 2π IV) 2π

(b) (3 points) $\lim\limits_{x \to 0^+} \dfrac{x^2 - 1}{x^3 - x}$

I) 1 II) $\dfrac{1}{2}$ III) ∞ IV) $-\infty$

(c) (3 points) $\lim\limits_{x \to \infty} \dfrac{3x^5 + x^3 + 1}{2x^4 + x^2 + x + 1}$

I) 0 II) $\dfrac{3}{2}$ III) ∞ IV) $-\infty$

(d) (3 points) $\lim\limits_{x \to \infty} \dfrac{\sqrt{x^2 - x + 2}}{5x}$

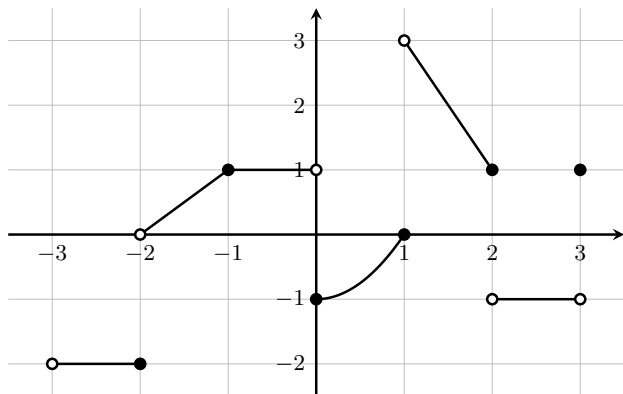I) 0 II) $\dfrac{1}{5}$ III) ∞ IV) $-\infty$

4. Compute the following limits.

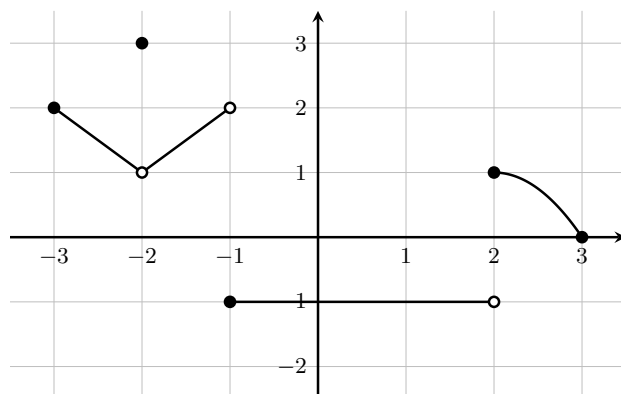   (a) (6 points) $\displaystyle\lim_{x\to 1} \frac{x^2 + 3x - 4}{x^2 + x - 2}$

   (b) (6 points) $\displaystyle\lim_{y\to 0} \frac{\sqrt{y + 16} - 4}{y}$

5. Use the following graphs to compute the given limits.

Graph of $f(x)$

Graph of $g(x)$

(a) (3 points) $\displaystyle\lim_{x\to 0^+} \left[\frac{f(x)}{g(x)}\right]$

(b) (3 points) $\displaystyle\lim_{x\to 1^-} f(g(x))$

(c) (3 points) $\displaystyle\lim_{x\to 0^+} g(f(x))$

(d) (3 points) $\displaystyle\lim_{x\to 2} \left[f(x) + g(x)\right]$

6. (13 points) Let $f(x) = 16x^{11} - 3x^2 - 12$. Use the intermediate Value Theorem to show that there exists a root c of $f(x)$ such that $0 < c < 1$. State any necessary assumptions.

7. (13 points) Evaluate the following limit. Show your work and cite any theorems you use.

$$\lim_{x \to \infty} \frac{\sin(x)}{x^2}$$

8. (13 points) Use the definition of the derivative to find $f'(1)$ if $f(x) = \dfrac{5}{x}$. (No credit if the definition is not used)