Derivative Practice: Inverse Trigonometric Functions

1. If $k(t) = 2^{\arcsin(\sqrt{t})}$, then what is $k'(t)$?

   Solution:

$$\begin{aligned} k'(t) &= (\ln 2)2^{\arcsin(\sqrt{t})} \cdot \frac{1}{\sqrt{1-(\sqrt{t})^2}} \cdot \frac{1}{2}t^{-\frac{1}{2}} \\ &= \frac{\ln 2}{2} \cdot 2^{\arcsin(\sqrt{t})} \cdot \frac{1}{\sqrt{t}\sqrt{1-t}} \end{aligned}$$

2. If $g(p) = \dfrac{p^2}{3}\arctan(5p-1) + k$, then what is $g'(p)$?

   Solution:

$$g'(p) = \frac{p^2}{3}\frac{1}{1+(5p-1)^2} \cdot 5 + \frac{2p}{3}\arctan(5p-1)$$

3. If $f(x) = \dfrac{x}{\arcsin(e^x)}$, then what is $f'(x)$?

   Solution:

$$f'(x) = \frac{\arcsin(e^x) - x \cdot \frac{e^x}{\sqrt{1-e^{2x}}}}{[\arcsin(e^x)]^2}$$

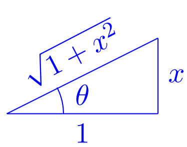
4. If $h(x) = \tan(\arctan(x))$, then what is $h'(x)$?

   Solution:

$$h'(x) = \frac{\sec^2(\arctan(x))}{1+x^2}$$

   Better solution: We have $h(x) = \tan(\arctan(x)) = x$ by inverse functions. So, $h'(x) = 1$.

   Note that by using the triangle technique, the first solution can be simplified:



$$\theta = \arctan(x)$$

$$\sec(\theta) = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{1+x^2}}{1}$$

$$\sec^2(\arctan(x)) = (\sec(\theta))^2 = 1 + x^2$$

$$\frac{\sec^2(\arctan(x))}{1+x^2} = \frac{1+x^2}{1+x^2} = 1$$

   Happily, the two methods of finding the derivative yield the same answer.