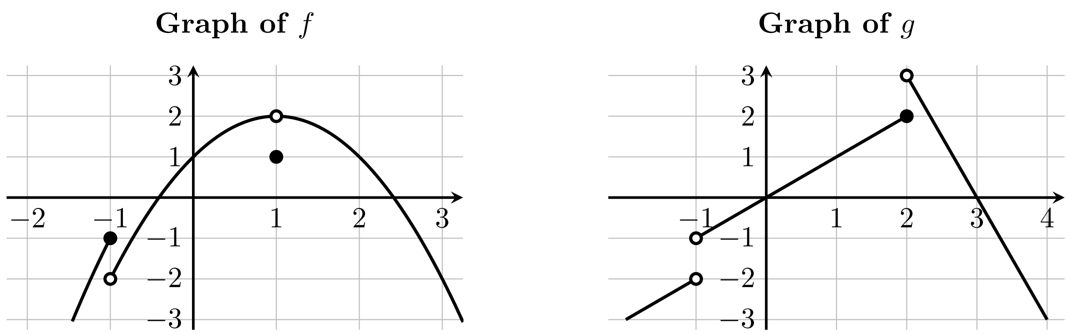
Graphical limits using limit laws **Name:** ____________________



1. $\lim_{x\to 0}(f(x)+g(x))$

   Solution: $\lim_{x\to 0}(f(x)+g(x)) = \lim_{x\to 0} f(x) + \lim_{x\to 0} g(x) = 1+0=1$

2. $\lim_{x\to 1}(f(x)g(x))$

   Solution: $\lim_{x\to 1}(f(x)g(x)) = \lim_{x\to 1} f(x) \cdot \lim_{x\to 1} g(x) = 2\cdot 1 = 2$

3. $\lim_{x\to 1}(f(x)+g(x))$

   Solution: $\lim_{x\to 1}(f(x)+g(x)) = \lim_{x\to 1} f(x) + \lim_{x\to 1} g(x) = 2+1=3$

4. $\lim_{x\to 2^+}(2f(x)+3g(x))$

   Solution: $\lim_{x\to 2^+}(2f(x)+3g(x)) = 2\cdot \lim_{x\to 2^+} f(x) + 3\cdot \lim_{x\to 2^+} g(x) = 2\cdot 1 + 3\cdot 3 = 11$

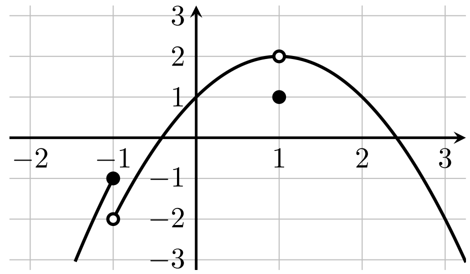
5. $\lim_{x\to 2^-}(x^2+(\ln x)\cdot g(x))$

   Solution: $\lim_{x\to 2^-}(x^2+(\ln x)\cdot g(x)) = \lim_{x\to 2^-} x^2 + \lim_{x\to 2^-}(\ln x \cdot g(x)) = 4 + \lim_{x\to 2^-} \ln x \cdot \lim_{x\to 2^-} g(x) = 4 + (\ln 2)\cdot 2 =$ $4+\ln 4$

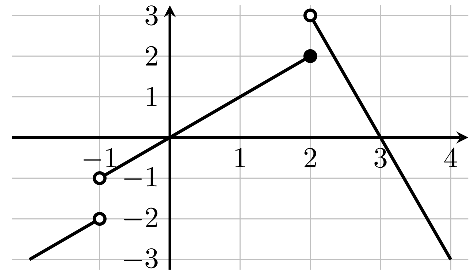
6. $\lim_{x\to 2}(f(x)-g(x))$

   Solution: First evaluate the two one-sided limits:
   $\lim_{x\to 2^+}(f(x)-g(x)) = \lim_{x\to 2^+} f(x) - \lim_{x\to 2^+} g(x) = 1-3=-2$
   $\lim_{x\to 2^-}(f(x)-g(x)) = \lim_{x\to 2^-} f(x) - \lim_{x\to 2^-} g(x) = 1-2=-1$
   We see that $\lim_{x\to 2^+}(f(x)-g(x)) \neq \lim_{x\to 2^-}(f(x)-g(x))$, so $\lim_{x\to 2}(f(x)-g(x))$ does not exist.

Graph of f



Graph of g



7. $\lim_{x\to 3} \dfrac{g(x)}{f(x)}$

Solution: $\lim_{x\to 3} \dfrac{g(x)}{f(x)} = \dfrac{\lim_{x\to 3} g(x)}{\lim_{x\to 3} f(x)} = \dfrac{0}{-2} = 0$

8. $\lim_{x\to 3^+} \dfrac{f(x)}{g(x)}$

Solution: We anticipate an issue because the limit of the denominator is 0, so we'll check the limits of the numerator and denominator separately. $\lim_{x\to 3^+} f(x) = -2$ and $\lim_{x\to 3^+} g(x) = 0$. The sign of the denominator is negative as x approaches 3 from the right. We have a non-zero limit divided by a number approaching 0 from below (which we can think of as the form "$\dfrac{-2}{0^-}$"), so $\lim_{x\to 3^+} \dfrac{f(x)}{g(x)} = +\infty$.

9. $\lim_{x\to 3} \dfrac{f(x)}{g(x)}$

Solution: From the previous problem, we know that we are dealing with a limit involving infinity, which tells us that we need to consider two one-sided limits. We already know that the limit from the right is $+\infty$, so next we'll look at the limit from the left. The limit of the numerator is $\lim_{x\to 3^-} f(x) = -2$ and the limit of the denominator is $\lim_{x\to 3^-} g(x) = 0$. This time the sign of the denominator is positive as x approaches 3 from the left. We have a non-zero limit divided by a number approaching 0 from below (which we can think of as the form "$\dfrac{-2}{0^+}$"), so $\lim_{x\to 3^-} \dfrac{f(x)}{g(x)} = -\infty$. Finally, $\lim_{x\to 3^+} \dfrac{f(x)}{g(x)} \neq \lim_{x\to 3-} \dfrac{f(x)}{g(x)}$, so $\lim_{x\to 3} \dfrac{f(x)}{g(x)}$ does not exist.

10. $\lim_{x\to 1} \sqrt{1 + f(x) + g(x)}$

Solution: $\lim_{x\to 1} \sqrt{1 + f(x) + g(x)} = \sqrt{\lim_{x\to 1}(1 + f(x) + g(x))} = \sqrt{1 + \lim_{x\to 1} f(x) + \lim_{x\to 1} g(x)} = \sqrt{1+2+1} = 2$

11. $\lim_{x\to -1} (f(x) + g(x))$

Solution: Since neither $\lim_{x\to -1} f(x)$ nor $\lim_{x\to -1} g(x)$ exists, we cannot use limit laws to break apart the limit. The jumps in both graphs at $x = -1$ hint to us to try two one-sided limits. Since these limits exist, we can then use the limit laws to break apart each of the one-sided limits.

$$\lim_{x\to -1^+} (f(x) + g(x)) = \lim_{x\to -1^+} f(x) + \lim_{x\to -1^+} g(x) = -2 + -1 = -3$$

$$\lim_{x\to -1^-} (f(x) + g(x)) = \lim_{x\to -1^-} f(x) + \lim_{x\to -1^-} g(x) = -1 + -2 = -3$$

Since $\lim_{x\to -1^+} (f(x) + g(x)) = \lim_{x\to -1^-} (f(x) + g(x)) = -3$, we have $\lim_{x\to -1} (f(x) + g(x)) = -3$. This is a little surprising - the jump discontinuities of the two functions manage to cancel each other out, and $f(x) + g(x)$ does have a limit at $x = -1$.