1. \( \lim_{x \to 0} (f(x) + g(x)) \)

Solution: \( \lim_{x \to 0} (f(x) + g(x)) = \lim_{x \to 0} f(x) + \lim_{x \to 0} g(x) = 1 + 0 = 1 \)

2. \( \lim_{x \to 1} (f(x)g(x)) \)

Solution: \( \lim_{x \to 1} (f(x)g(x)) = \lim_{x \to 1} f(x) \cdot \lim_{x \to 1} g(x) = 2 \cdot 1 = 2 \)

3. \( \lim_{x \to 1} (f(x) + g(x)) \)

Solution: \( \lim_{x \to 1} (f(x) + g(x)) = \lim_{x \to 1} f(x) + \lim_{x \to 1} g(x) = 2 + 1 = 3 \)

4. \( \lim_{x \to 2^+} (2f(x) + 3g(x)) \)

Solution: \( \lim_{x \to 2^+} (2f(x) + 3g(x)) = 2 \cdot \lim_{x \to 2^+} f(x) + 3 \cdot \lim_{x \to 2^+} g(x) = 2 \cdot 1 + 3 \cdot 3 = 11 \)

5. \( \lim_{x \to 2^-} (x^2 + (\ln x) \cdot g(x)) \)

Solution: \( \lim_{x \to 2^-} (x^2 + (\ln x) \cdot g(x)) = \lim_{x \to 2^-} x^2 + \lim_{x \to 2^-} (\ln x \cdot g(x)) = 4 + \lim_{x \to 2^-} \ln x \cdot \lim_{x \to 2^-} g(x) = 4 + (\ln 2) \cdot 2 = 4 + \ln 4 \)

6. \( \lim_{x \to 2^-} (f(x) - g(x)) \)

Solution: First evaluate the two one-sided limits:
\[
\lim_{x \to 2^-} (f(x) - g(x)) = \lim_{x \to 2^-} f(x) - \lim_{x \to 2^-} g(x) = 1 - 3 = -2
\]
\[
\lim_{x \to 2^+} (f(x) - g(x)) = \lim_{x \to 2^+} f(x) - \lim_{x \to 2^+} g(x) = 1 - 2 = -1
\]
We see that \( \lim_{x \to 2^-} (f(x) - g(x)) \neq \lim_{x \to 2^+} (f(x) - g(x)) \), so \( \lim_{x \to 2} (f(x) - g(x)) \) does not exist.
7. \( \lim_{x \to 3} \frac{g(x)}{f(x)} \)

**Solution:** \( \lim_{x \to 3} \frac{g(x)}{f(x)} = \lim_{x \to 3} g(x) / \lim_{x \to 3} f(x) = 0 / -2 = 0 \)

8. \( \lim_{x \to 3^+} \frac{f(x)}{g(x)} \)

**Solution:** We anticipate an issue because the limit of the denominator is 0, so we’ll check the limits of the numerator and denominator separately. \( \lim_{x \to 3^+} f(x) = -2 \) and \( \lim_{x \to 3^+} g(x) = 0 \). The sign of the denominator is negative as \( x \) approaches 3 from the right. We have a non-zero limit divided by a number approaching 0 from below (which we can think of as the form \( \frac{-2}{0^-} \)), so \( \lim_{x \to 3^+} g(x) = +\infty \).

9. \( \lim_{x \to 3^+} f(x) \)

**Solution:** From the previous problem, we know that we are dealing with a limit involving infinity, which tells us that we need to consider two one-sided limits. We already know that the limit from the right is \( +\infty \), so next we’ll look at the limit from the left. The limit of the numerator is \( \lim_{x \to 3^-} f(x) = -2 \) and the limit of the denominator is \( \lim_{x \to 3^-} g(x) = 0 \). This time the sign of the denominator is positive as \( x \) approaches 3 from the left. We have a non-zero limit divided by a number approaching 0 from below (which we can think of as the form \( \frac{-2}{0^+} \)), so \( \lim_{x \to 3^-} g(x) = -\infty \). Finally, \( \lim_{x \to 3^+} f(x) \neq \lim_{x \to 3^-} f(x) \), so \( \lim_{x \to 3} f(x) \) does not exist.

10. \( \lim_{x \to 1} \sqrt{1 + f(x) + g(x)} \)

**Solution:** \( \lim_{x \to 1} \sqrt{1 + f(x) + g(x)} = \sqrt{\lim_{x \to 1} (1 + f(x) + g(x))} = \sqrt{1 + \lim_{x \to 1} f(x) + \lim_{x \to 1} g(x)} = \sqrt{1 + 2 + 1} = 2 \)

11. \( \lim_{x \to -1} (f(x) + g(x)) \)

**Solution:** Since neither \( \lim_{x \to -1} f(x) \) nor \( \lim_{x \to -1} g(x) \) exists, we cannot use limit laws to break apart the limit. The jumps in both graphs at \( x = -1 \) hint to us to try two one-sided limits. Since these limits exist, we can then use the limit laws to break apart each of the one-sided limits.

\[
\lim_{x \to -1^+} (f(x) + g(x)) = \lim_{x \to -1^+} f(x) + \lim_{x \to -1^+} g(x) = -2 + -1 = -3
\]

\[
\lim_{x \to -1^-} (f(x) + g(x)) = \lim_{x \to -1^-} f(x) + \lim_{x \to -1^-} g(x) = -1 + -2 = -3
\]

Since \( \lim_{x \to -1^+} (f(x) + g(x)) = -3 \), we have \( \lim_{x \to -1} (f(x) + g(x)) = -3 \). This is a little surprising - the jump discontinuities of the two functions manage to cancel each other out, and \( f(x) + g(x) \) does have a limit at \( x = -1 \).