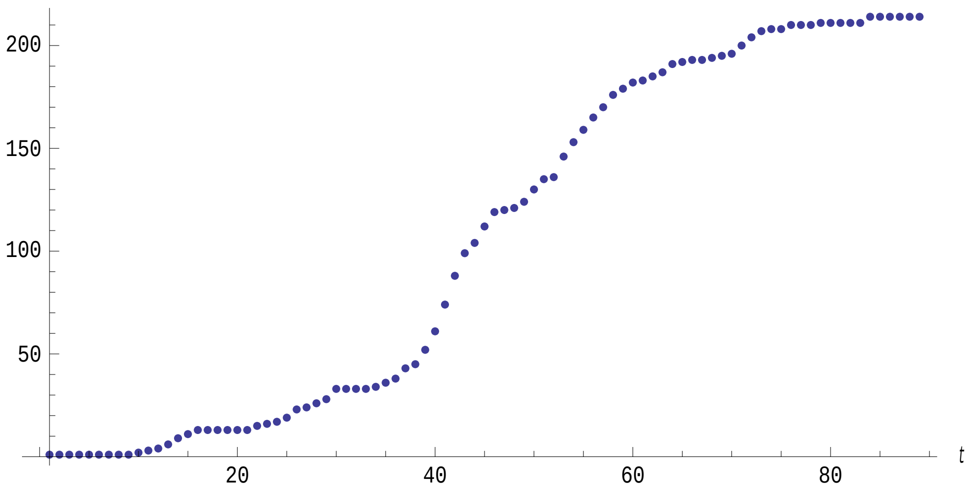
Modeling with an inverse trig function – an Ebola outbreak in 1995

Goal: To study a model for deaths due to an Ebola outbreak and to use this model to discover a connection between the values of a function and the area under the function's derivative.

In 1995 there was a 90-day-long outbreak of Ebola in the Democratic Republic of Congo (DRC). The points below are a plot of a function $N(t)$ which represents the total number of deaths from the beginning of the outbreak to the end of day t.



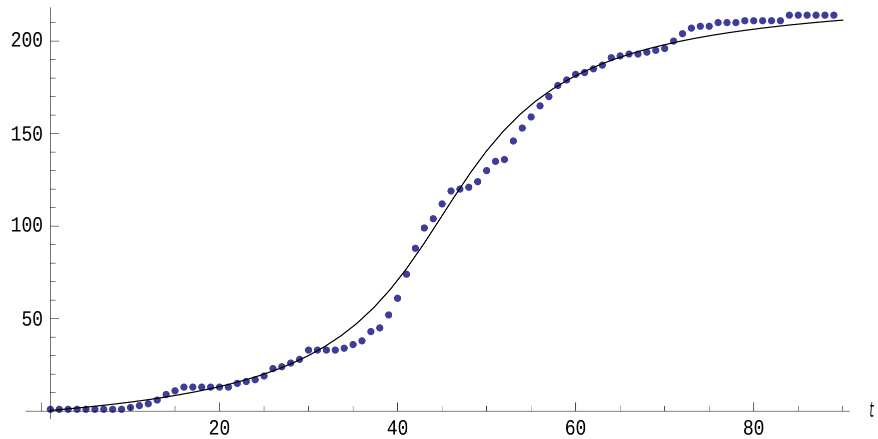
1. (a) Translate the equation $N(22) = 15$ into an explanatory English sentence.

   (b) The data shows that $13 = N(16) = N(17) = N(18) = N(19) = N(20) = N(21)$. What can you conclude from this data? (Give your answer in a complete English sentence.)

An important way to analyze data is to find a function that models the data–which means that the graph of the function closely fits the data points. The function

$$D(t) = \frac{1654}{21}\left(\arctan\left(\frac{2(t-45)}{21}\right) + \arctan\left(\frac{30}{7}\right)\right)$$

is a good model for the 1995 Ebola data as the below graph shows:

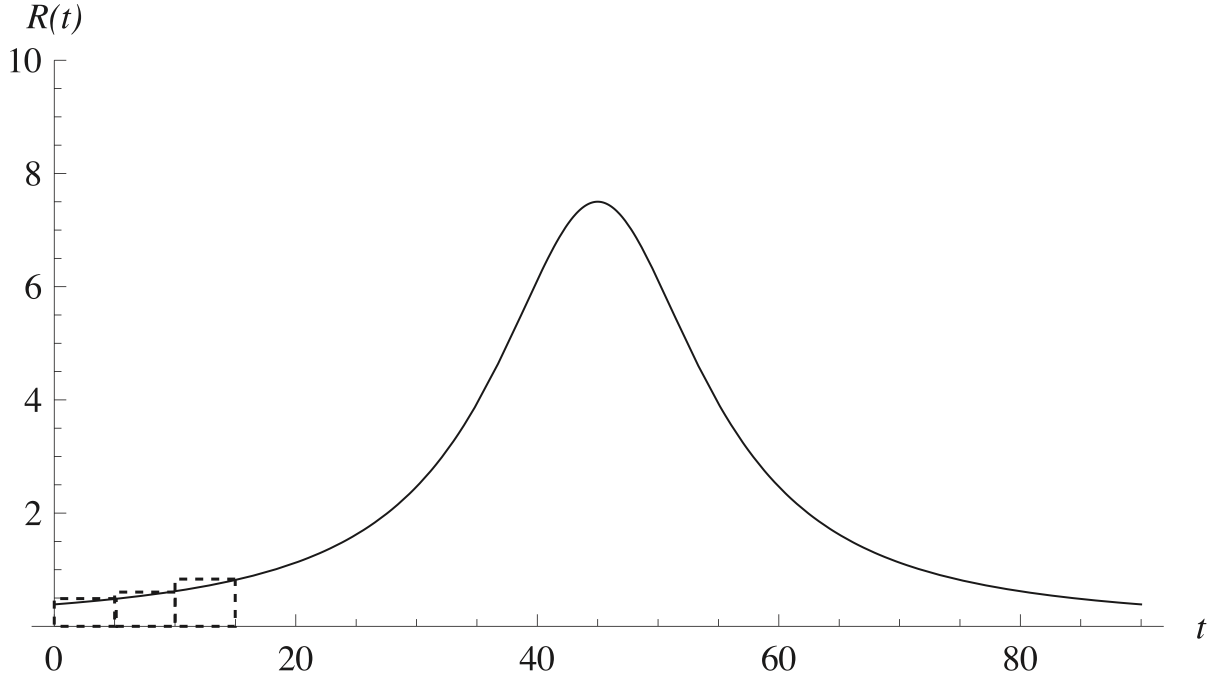


2. How well does the *mathematical model* $D(t)$ for the number of deaths represent the *actual* cumulative death count $N(t)$? When does the model least accurately reflect the data? When do we see the largest discrepancy between the rate of change of the model and the rate of change of the actual data?

3. Assuming, for the moment, that the actual cumulative death count is given by the above function $D(t)$, show that the instantaneous death rate $R(t)$, in deaths per day, is given by the formula

$$R(t) = \frac{3308}{441 + 4(t-45)^2}.$$

The function $R(t)$ that you found in problem 3 is sketched on the axes below. (Note the bell shape!!)



Note that we've dashed in a rectangle over each of the first three intervals of length 5, on the above t axis. The height of each rectangle is just the value of the function $R(t)$ at the right endpoint of the interval in question.

4. Continue the process of drawing rectangles over each of the above subintervals of length 5, on the t axis, with the height of each rectangle being the value of $R(t)$ at the rightmost edge of the rectangle.

5. Let $T(t_0)$ denote the *total area* of the rectangles you've sketched in, between $t = 0$ and $t = t_0$. Compute $T(20)$, $T(40)$, $T(60)$, and $T(80)$. (Recall that each rectangle has base length 5, and height given by the value of $R(t)$ at the right endpoint of the rectangle.) (The total areas you are considering here are called *Riemann sums*; more on these later in class.)

6. Compare the above numbers $T(20)$, $T(40)$, $T(60)$, and $T(80)$, to the numbers $D(20)$, $D(40)$, $D(60)$, and $D(80)$ you get by plugging in the appropriate t-values into the formula for $D(t)$ above. Do you see a correspondence between these sequences of numbers? Do you have any idea why this correspondence should be true?

   (This correspondence amounts to a HUGE theorem, called the Fundamental Theorem of Calculus, which we'll discuss in class later.)

   Why do you think the values of $T(t)$ that you calculated in problem 5 start off higher than corresponding values of $D(t)$, and then later $D(t)$ catches up?