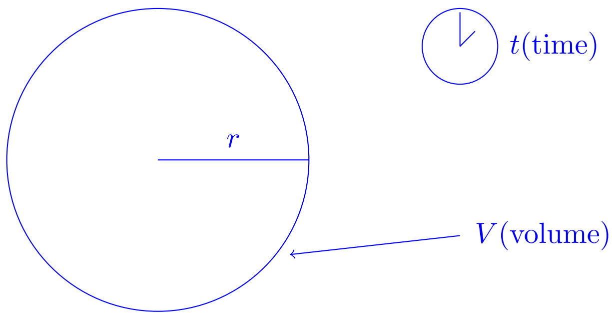
1. A spherical snowball melts in such a way that the instant at which its radius is 20 cm, its radius is decreasing at 3 cm/min. At what rate is the volume of the ball of snow changing at that instant?

   (a) Diagram: Draw a picture of the melting snowball. Label the variables of interest.

   Solution:

   (b) Rates: What is the known rate of change? What is the needed rate of change? Include units.

   Solution: The known rate of change is $\frac{dr}{dt} = -3\frac{\text{cm}}{\text{min}}$, and the unknown rate of change is $\frac{dV}{dt}$, units $\frac{\text{cm}^3}{\text{min}}$.

   (c) Equation: The rates in the previous part involved the variables V and r. Write an equation from geometry relating V and r.

   Solution: $V = \frac{4}{3}\pi r^3$

   (d) Differentiate: because the snowball is melting, both the radius and volume are really functions of time. Differentiate your formula from the last part with respect to time, t, in minutes.

   Solution: $\frac{dV}{dt} = \frac{4}{3}\pi 3r^2\frac{dr}{dt} = 4\pi r^2\frac{dr}{dt}$

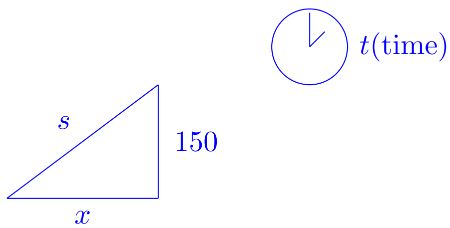
   (e) Substitute and solve: Plug all known quantities into your equation from the last part and solve for the desired rate. Answer the question asked.

   Solution: $\frac{dV}{dt} = 4\pi \cdot 20^2 \cdot -3\left(\frac{\text{cm}^3}{\text{min}}\right)$

2. Omar flies his kite 150m high, where the wind causes it to move horizontally away from him at the rate of 5m per second. In order to maintain the kite at a height of 150m, Omar must allow more string to be let out. At what rate is the string being let out when the length of the string already out is 250m?

   (a) Diagram:

   Solution:

   (b) Rates:

   Solution: The known rate is $\frac{dx}{dt}$ and the unknown rate is $\frac{ds}{dt}$.

   (c) Equation:

   Solution: $s^2 = 150^2 + x^2$

   (d) Differentiate:

   Solution: $2s\frac{ds}{dt} = 2x\frac{dx}{dt}$

   (e) Substitute:

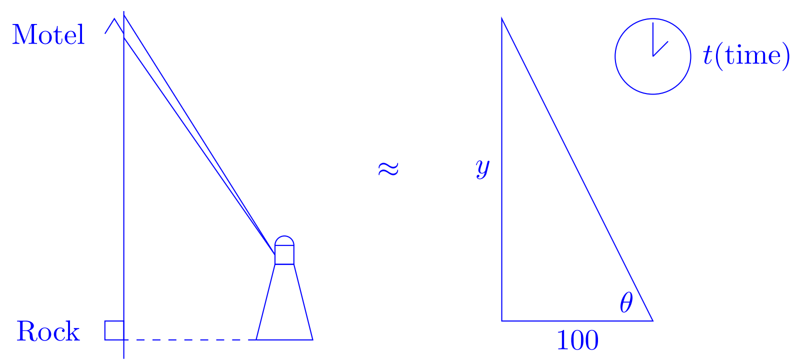
   Solution: At the time in question, $s = 250$. To get x at this instant, $250^2 = 150^2 + x^2$, so $x = 200$m. Substituting into the equation from the last part:
   $2 \cdot 250\frac{ds}{dt} = 2 \cdot 200 \cdot 5$

   (f) Solve:

   Solution: $\frac{ds}{dt} = \frac{200 \cdot 5}{250}\left(\frac{\mathrm{m}}{\mathrm{s}}\right) = 4\frac{\mathrm{m}}{\mathrm{s}}$.

3. On the shore sits Sea Lion Rock. A lighthouse stands off-shore, 100 yards east of Sea Lion Rock. 173 yards due north of Sea Lion Rock is the exclusive See Lion Motel. The lighthouse light rotates twice a minute. At the moment the beam of light hits the motel, how fast is the beam of light moving along the coast?

Solution:



Since the beam of light makes two full circles per minute, $\frac{d\theta}{dt} = 2 \cdot 2\pi \left(\frac{\text{rad}}{\text{min}}\right)$. The rate we want to know is $\frac{dy}{dt}$.

$$\tan(\theta) = \frac{y}{100}$$
$$\sec^2(\theta)\frac{d\theta}{dt} = \frac{1}{100}\frac{dy}{dt}$$
$$\frac{dy}{dt} = 100\sec^2(\theta)\frac{d\theta}{dt}$$

Using the Pythagorean Theorem, the hypotenuse of the triangle is about 200 meters. So $\sec(\theta) = \frac{\text{hypotenuse}}{\text{adjacent}} \approx \frac{200}{100} = 2$.

$$\frac{dy}{dt} \approx 100 \cdot 2^2 \cdot 4\pi \left(\frac{\text{yards}}{\text{min}}\right) = 1600\pi\frac{\text{yards}}{\text{min}}$$