1. Let $f(x) = (3x^2 + 1)^2$. We are going to find the derivative of $f(x)$ in three ways and then compare the answers.

   (a) Algebraically multiply out the expression for $f(x)$ and then take the derivative.

   Solution:
   $$f(x) = (3x^2 + 1)^2 = 9x^4 + 6x^2 + 1 \text{ so } f'(x) = 36x^3 + 12x$$

   (b) View $f(x)$ as a product of two functions, $f(x) = (3x^2 + 1)(3x^2 + 1)$ and use the product rule to find $f'(x)$.

   Solution: Let $u = 3x^2 + 1$ and $v = 3x^2 = 1$, then $(uv)' = u'v + v'u = (6x)(3x^2 + 1) + (6x)(3x^2 + 1) = 36x^3 + 12x$

   (c) Apply the chain rule directly to the expression $f(x) = (3x^2 + 1)^2$

   Solution: $f'(x) = 2(3x^2 + 1)(6x) = 36x^3 + 12x$

   (d) Are your answers in parts a, b, and c the same? Why or why not?

   Solution: All the answers are the same because it doesn't matter which method you use to take a derivative. If done correctly, they should all give the same answer.

2. Let $f(x) = \sin(2x)$. We are going to find the derivative of $f(x)$ in two ways and then compare answers. You will need the double angle formulas for this problem:

   - $\sin 2x = 2 \sin x \cos x$
   - $\cos 2x = \cos^2 x - \sin^2 x$

   (a) Rewrite $\sin(2x)$ using the double-angle formula, then apply the product rule to find $f'(x)$.

   Solution: $f(x) = \sin 2x = 2\sin(x)\cos(x)$, by the sine double angle-formula. Let $u = 2\sin(x)$ and $v = \cos(x)$, then $f'(x) = 2\cos(x)\cos(x) - 2\sin(x)\sin(x) = 2(\cos^2(x) - \sin^2(x))$

   (b) Apply the chain rule directly to the expression $f(x) = \sin(2x)$ to find its derivative a second way.

   Solution: $f'(x) = \cos(2x) \cdot 2 = 2\cos(2x)$

   (c) Are your answers in parts a and b the same? Why or why not?

   Solution: Part (a) gives $= f'(x) = 2(\cos^2(x) - \sin^2(x))$, by the cosine double-angle formula, $f'(x) = 2\cos(2x)$. The two answers are the same because it doesn't matter which method you use to take a derivative. If done correctly, they should all give the same answer.

3. Suppose f is differentiable and that $g(x) = (f(\sqrt{x}))^3$.

   (a) Calculate $g'(x)$ (your answer will include f and f').

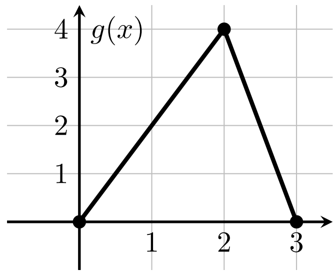
   Solution: Applying the chain rule twice we get $g'(x) = 3(f(\sqrt{x}))^2 f'(\sqrt{x}) \frac{1}{2\sqrt{x}}$.

   (b) If $f(2) = 1$ and $f'(2) = -2$, calculate $g'(4)$.

   Solution: $g'(4) = 3(f(\sqrt{4}))^2 f'(\sqrt{4}) \frac{1}{2\sqrt{4}} = 3(f(2))^2 f'(2) \frac{1}{4} = 3 \cdot 1^2 \cdot -2 \cdot \frac{1}{4} = -\frac{3}{2}$.

4. Let $f(x)$ and $g(x)$ be two functions. Values of $f(x)$ and $f'(x)$ are given in the table below and the graph of $g(x)$ is as shown.

x	1	2	3
$f(x)$	3	2	1
$f'(x)$	4	5	6



(a) Let $h(x) = g(f(x))$. Find $h'(3)$.

Solution: $h'(3) = g'(f(3)) \cdot f'(3) = g'(1) \cdot f'(3) = 2 \cdot 6 = 12$

(b) Let $k(x) = f(g(x))$. Find $k'(1)$.

Solution: $k'(1) = f'(g(1)) \cdot g'(1) = f'(2) \cdot g'(1) = 5 \cdot 2 = 10.$

5. The US population on July 1 of 2010 was 309.33 million. The population was 311.59 million on July 1 of 2011.

(a) Find an exponential model $p(t)$ to fit this data. Let $t = 0$ on July 1, 2010.

Solution: We're looking for a function of the form $p(t) = Ae^{kt}$, in millions of people. Substituting $p(0) = 309.33$, we see that $A = 309.33$. Substituting $p(1) = 311.59$ gives $311.59 = 309.33 \cdot e^k$. Solving gives $e^k = 311.59/309.33$, so $k = \ln(311.59/309.33) \approx .00728$. This is an annual growth rate of .728%. Our model is $p(t) = 309.33 \cdot e^{.00728t}$.

(b) Use your model to estimate the US population on November 1 of 2013.

Solution: Substituting $t = 3.33$ into $p(t) = 309.33e^{.00728t}$ gives $p(3) \approx 316.92$ million people. The actual value was approximately 316.98 million.

(c) Find $p'(3)$. Interpret the meaning of this number, including units.

Solution: First take the derivative: $p'(t) = .00728 \cdot 309.33e^{.00728t}$. Substituting $t = 3$ gives $p'(3) = .00728 \cdot 309.33e^{.00728 \cdot 3} \approx 2.3$. This means that on July 1 in the year 2013 the rate of change of the US population was approximately 2.3 million people per year.

6. Chains, Inc. is in the business of making and selling chains. Let $c(t)$ be the number of miles of chain produced after t hours of production. Let $p(c)$ be the profit as a function of the number of miles of chain produced, and let $q(t)$ be the profit as a function of the number of hours of production.

   (a) Suppose the company can produce three miles of chain per hour, and suppose their profit on the chains is \$4000 per mile of chain. Find each of the following (include units).

   $c(t) = 3t$ miles

   $c'(t) = 3$ miles/hour

   Meaning of $c'(t)$: 3 feet of chain are produced per hour.

   $p(c) = 4000c$ dollars

   $p'(c) = 4000$ dollars/mile

   Meaning of $p'(c)$: 4000 dollars of profit are earned per mile of chain produced.

   $q(t) = 12000t$ dollars

   $q'(t) = 12000$ dollars/hour

   Meaning of $q'(t)$: 12000 dollars of profit are earned per hour of production.

   How does $q'(t)$ relate to $p'(c)$ and $c'(t)$?
   Solution: By the chain rule $q'(t) = p'(c(t))c'(t)$. So $q'(t) = 4000$ dollars/mile $\cdot 3$ miles/hour $= 12000$ dollars/hours.

   (b) In this part, the production and profit functions are no longer linear. Instead $p(c)$ is modeled by the formula $p(c) = 100 - 100\cos\left(\frac{\pi}{38}c\right)$ (where p is measured in thousands of dollars and c is measured in miles of chain), and $c(t)$ is defined numerically below:

t (in hours)	2	4	6	8	10
c (in miles)	6	14	24	38	52

   Estimate $q'(4)$ and $q'(8)$. What conclusions should you draw about production?

   Solution: First note that $p'(c) = \frac{100\pi}{38}\sin\left(\frac{\pi}{38}c\right)$.
   Using the chain rule, $q'(4) = p'(c(4))c'(4)$. Estimating numerically, $c'(4) \approx \frac{24-6}{6-2} = \frac{18}{4}$.
   So $q'(4) \approx \frac{100\pi}{38}\sin\left(\frac{\pi}{38}\cdot 14\right)\cdot\frac{18}{4} \approx 34.07$ thousand dollars/hour. This means that after 4 hours of production, the profit increases a rate of about \$34,000 dollars per added hour of production. We should keep the factory running. Similarly, we find that $q'(8) = p'(c(8))c'(8) \approx \frac{100\pi}{38}\sin\left(\frac{\pi}{38}\cdot 38\right)\cdot\frac{28}{4} \approx 0$. The profit is no longer increasing as we increase the number of hours of production. We should determine if this is a maximum, and possibly shut down the factory after 8 hours.