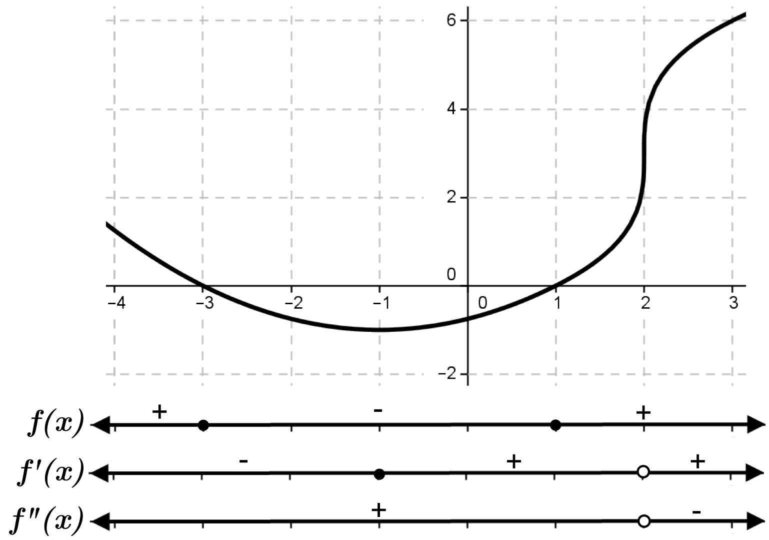
1. In the graph of $f(x)$ below, number lines are used to mark where $f(x)$ is zero/positive/negative/undefined, where $f'(x)$ is zero/positive/negative/undefined, and where $f''(x)$ is zero/positive/negative/undefined. Closed circles on the number lines indicate a zero-value, and open-circles indicate an undefined value.



   (a) What do the closed circles on the number line for $f(x)$ correspond to on the graph of $f(x)$?

   (b) How does each + or $-$ sign on the number line for $f(x)$ relate to the graph?

   (c) What does the closed circle at $x = -1$ on the number line for $f'(x)$ correspond to on the graph of $f(x)$?

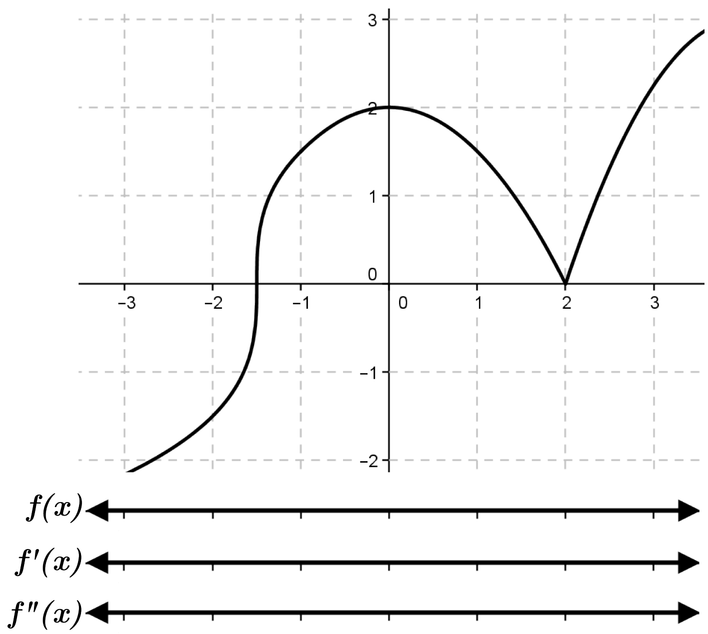
   (d) What does the open circle at $x = 2$ on the number line for $f'(x)$ correspond to on the graph of $f(x)$?

   (e) How does each + or $-$ sign on the number line for $f'(x)$ relate to the graph?

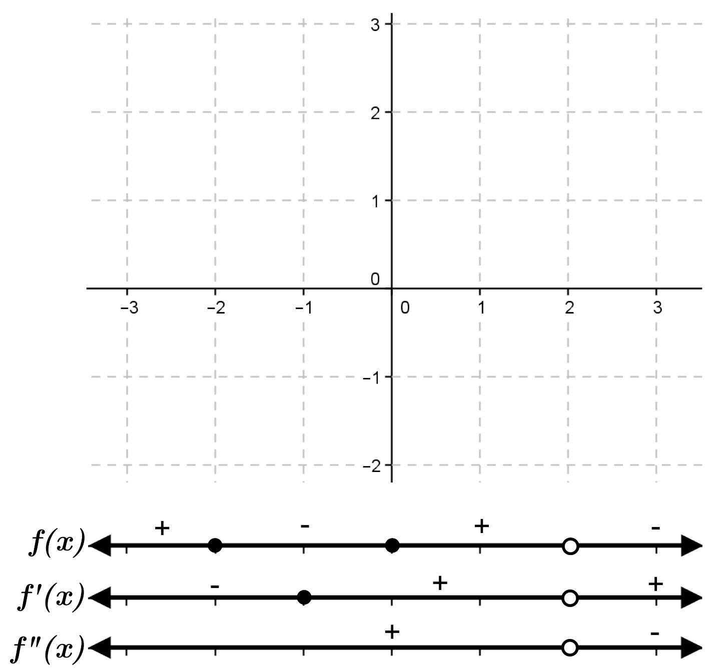
   (f) What does the open circle at $x = 2$ on the number line for $f''(x)$ correspond to on the graph of $f(x)$?

   (g) How does each + or $-$ sign on the number line for $f''(x)$ relate to the graph?

2. For the graph of $f(x)$ shown below, fill in the number lines for $f(x)$, $f'(x)$ and $f''(x)$, marking closed circles where there is a zero, marking open circles for undefined points, and marking $+$ and $-$ signs on each interval to show positive/negativeness.

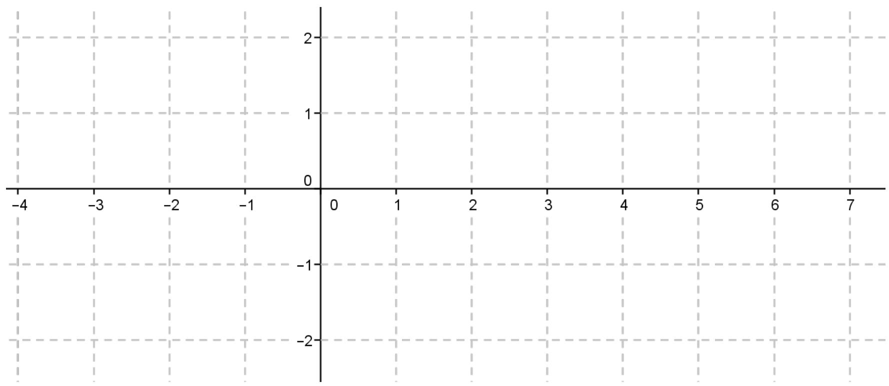


3. Draw a graph of $f(x)$ that fits the information shown in the number lines.

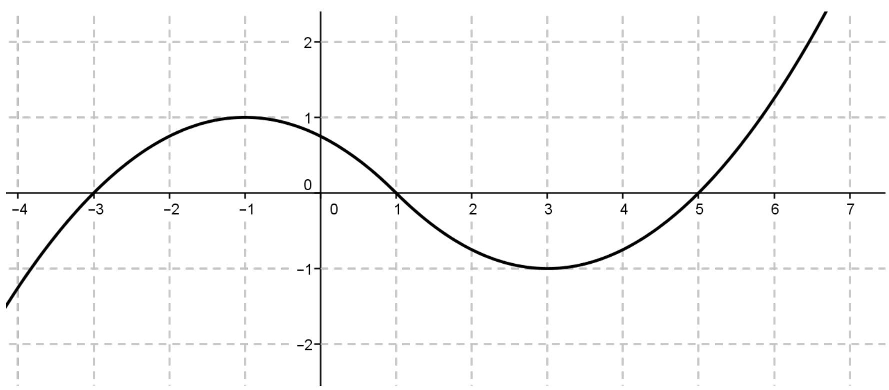


4. The middle graph drawn below shows $f'(x)$. Using the principles you learned in the previous problem, draw a possible graph of $f(x)$ above it, and a graph of $f''(x)$ below it. (If you are stuck try drawing the graph of $f''(x)$ first.)

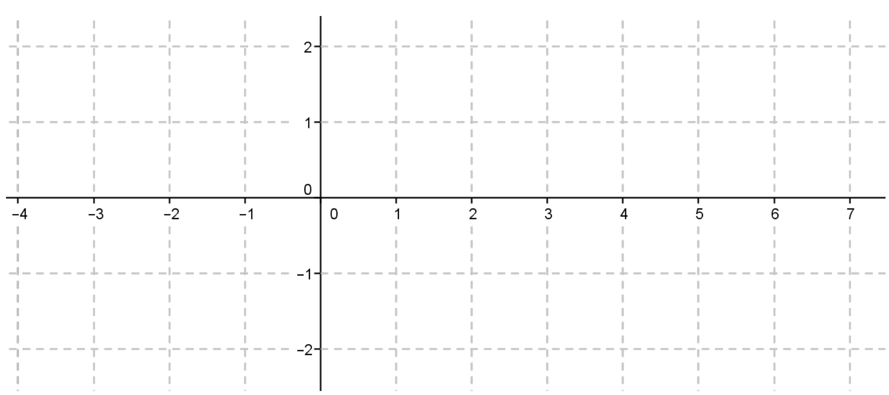
$f(x)$:



$f'(x)$:



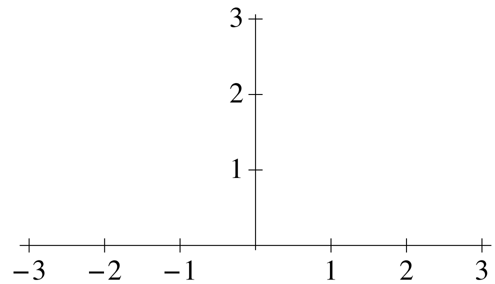
$f''(x)$:



5. This problem investigates the derivative of the absolute value function. Recall that we define the absolute value as:

$$|x| = \begin{cases} x & \text{if } x \geq 0, \\ -x & \text{if } x < 0. \end{cases}$$

(a) In the space provided, draw a graph of the function $f(x) = |x|$.



(b) Using your graph from part (a), and your understanding of the derivative as the rate of change/slope of the tangent line, find the derivative function $f'(x)$ of the above function $f(x) = |x|$, for x not equal to 0 (fill in the blanks):

$$f'(x) = \begin{cases} \_\_\_\_\_\_\_\_\_\_\_\_ & \text{if } x > 0, \\ \_\_\_\_\_\_\_\_\_\_\_\_ & \text{if } x < 0. \end{cases}$$

.

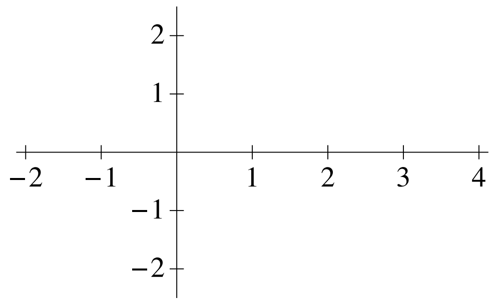
(c) But what about $f'(0)$? It is not so clear from the picture even how to draw a tangent line to the function at the origin. So let's try to compute $f'(0)$ by first looking at the corresponding lefthand and righthand limits of the difference quotient.

i. Compute $\lim_{h \to 0^+} \frac{f(0+h)-f(0)}{h}$. Hint: use the piecewise definition of $f(x)$ given above.

ii. Compute $\lim_{h \to 0^-} \frac{f(0+h)-f(0)}{h}$.

iii. What do your answers to parts (i) and (ii) tell you about $f'(0)$? Please explain.

6. Using what you've learned above, sketch the graph of a *continuous* function $g(x)$ such that $g(x)$ is not differentiable at $x = -1$, $x = 2$, nor $x = 3$.



7. Create a piecewise function where one piece is a quadratic function and the other piece is a linear function which is continuous everywhere but not differentiable at $x = 0$.

$$f(x) = \begin{cases} \underline{\hspace{3cm}} & \text{if } x \geq 0, \\ \underline{\hspace{3cm}} & \text{if } x < 0. \end{cases}$$

.

8. Create a piecewise function where one piece is a quadratic function and the other piece is a linear function which is continuous and differentiable everywhere.

$$f(x) = \begin{cases} \underline{\hspace{3cm}} & \text{if } x > 0, \\ \underline{\hspace{3cm}} & \text{if } x \leq 0. \end{cases}$$

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