

1. Calculate the derivative (slope of the tangent line) using the definition.

<p>Distribute/FOIL:</p> $f'(3) = \lim_{h \rightarrow 0} \frac{9 + 6h + h^2 + 15 + 5h - 24}{h}$	<p>Step: <u>4</u></p>
<p>Collect like terms:</p> $f'(3) = \lim_{h \rightarrow 0} \frac{h^2 + 11h}{h}$	<p>Step: <u>5</u></p>
<p>Evaluate the limit:</p> $f'(3) = 11$	<p>Step: <u>8</u></p>
<p>Definition of the derivative at $x = 3$:</p> $f'(3) = \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$	<p>Step: <u>1</u></p>
<p>Factor:</p> $f'(3) = \lim_{h \rightarrow 0} \frac{h(h + 11)}{h}$	<p>Step: <u>6</u></p>
<p>Begin Simplifying:</p> $f'(3) = \lim_{h \rightarrow 0} \frac{(3+h)(3+h) + 5(3+h) - 24}{h}$	<p>Step: <u>3</u></p>
<p>Cancel:</p> $f'(3) = \lim_{h \rightarrow 0} h + 11$	<p>Step: <u>7</u></p>
<p>Use $f(x) = x^2 + 5x$</p> $f'(3) = \lim_{h \rightarrow 0} \frac{(3+h)^2 + 5(3+h) - 24}{h}$	<p>Step: <u>2</u></p>

2. $f(x) = 3x^2 - x$. Find $f'(-2)$ by circling the correct next step from each row. In the third column, explain why this is the correct step and what caused the error in the incorrect step.

Step 1.	$\lim_{h \rightarrow 0} \frac{f(-2+h) - f(-2)}{h}$	$\lim_{h \rightarrow -2} \frac{f(-2+h) - f(-2)}{h}$	The definition of derivative has $h \rightarrow 0$.
Step 2.	$\lim_{h \rightarrow 0} \frac{3(-2+h)^2 + 2 + h + 14}{h}$	$\lim_{h \rightarrow 0} \frac{3(-2+h)^2 + 2 - h - 14}{h}$	The numerator should be $3(-2+h)^2 - (-2+h) - 14$. In left column the negative is not distributed, and then there is another sign error with $f(-2)$.
Step 3.	$\lim_{h \rightarrow 0} \frac{3(4 - 4h + h^2) + 2 - h - 14}{h}$	$\lim_{h \rightarrow 0} \frac{3(4 + h^2) - 2 - h - 14}{h}$	$(-2+h)^2 = (-2+h)(-2+h) = 4 - 4h + h^2$. Algebra error in the right column - it was not distributed correctly.
Step 4.	$\lim_{h \rightarrow 0} \frac{12 - 4h + h^2 + 2 - h - 14}{h}$	$\lim_{h \rightarrow 0} \frac{12 - 12h + 3h^2 + 2 - h - 14}{h}$	The 3 wasn't distributed to all three terms in the left column.
Step 5.	$\lim_{h \rightarrow 0} \frac{-12h + 3h^2 - h}{h}$	$\lim_{h \rightarrow 0} \frac{3h^2 - 13h + 12}{h}$	In the right column the constants were added wrong. The left column is correct, although not fully simplified yet.
Step 6.	$\lim_{h \rightarrow 0} \frac{-12\cancel{h} + 3h^2 - h}{\cancel{h}}$	$\lim_{h \rightarrow 0} \frac{3h^2 - 13h}{h}$	The left column has a serious algebra error - no cancellation unless the h is factored out first.
Step 7.	$\frac{3h^2 - 13h}{h}$	$\lim_{h \rightarrow 0} \frac{h(3h - 13)}{h}$	The left column is missing a limit.
Step 8.	$\lim_{h \rightarrow 0} (3h - 13)$	$3h - 13h$	The right column is missing a limit. Also, it looks like they used the wrong answer from Step 7, and then incorrectly simplified.
Step 9.	$-10h$	-13	The left column is a (correct) simplification of the wrong choice for Step 8.

3. $f(x) = \sqrt{x}$. Find $f'(4)$ by filling in the boxes with the correct mathematical expressions.

$$f'(4) = \lim_{h \rightarrow 0} \frac{f(\boxed{4+h}) - f(\boxed{4})}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\boxed{\sqrt{4+h} - \sqrt{4}}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{4+h} - 2}{h} \cdot \frac{\sqrt{4+h} + 2}{\sqrt{4+h} + 2}$$

$$= \lim_{h \rightarrow 0} \frac{\boxed{4+h+2\sqrt{4+h}-2\sqrt{4+h}-4}}{h(\sqrt{4+h}+2)}$$

$$= \lim_{h \rightarrow 0} \frac{4+h+\boxed{-4}}{h(\sqrt{4+h}+2)}$$

$$= \lim_{h \rightarrow 0} \frac{\boxed{h}}{h(\sqrt{4+h}+2)}$$

$$= \lim_{h \rightarrow 0} \frac{1}{\sqrt{4+h}+2}$$

$$= \frac{1}{\sqrt{4+\boxed{0}}+2}$$

$$= \boxed{\frac{1}{4}}$$

4. Now you're on your own! Use the definition of the derivative to calculate the following derivatives.

(a) $f(x) = 7x^2 + 5x$. Find $f'(2)$.

Definition of derivative at $x = 2$: $f'(2) = \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h}$

Use the function $f(x) = 7x^2 + 5x$: $= \lim_{h \rightarrow 0} \frac{(7(2+h)^2 + 5(2+h)) - (7 \cdot 2^2 + 5 \cdot 2)}{h}$

FOIL and simplify: $= \lim_{h \rightarrow 0} \frac{(7(4 + 4h + h^2) + 10 + 5h) - (28 + 10)}{h}$

Distribute and simplify: $= \lim_{h \rightarrow 0} \frac{28 + 28h + 7h^2 + 10 + 5h - 38}{h}$

Collect constant terms: $= \lim_{h \rightarrow 0} \frac{28h + 7h^2 + 5h}{h}$

Collect all like terms: $= \lim_{h \rightarrow 0} \frac{7h^2 + 33h}{h}$

Factor: $= \lim_{h \rightarrow 0} \frac{h(7h + 33)}{h}$

Cancel: $= \lim_{h \rightarrow 0} 7h + 33$

Evaluate the limit: $= 7(0) + 33$

$$= 33.$$

(b) $f(x) = \frac{1}{x}$. Find $f'(3)$.

Definition of derivative at $x = 3$: $f'(3) = \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$

Use the function $f(x) = \frac{1}{x}$: $= \lim_{h \rightarrow 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h}$

Clear mini-denominators (optional): $= \lim_{h \rightarrow 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h} \cdot \frac{3(3+h)}{3(3+h)}$

Distribute: $= \lim_{h \rightarrow 0} \frac{\frac{3(3+h)}{3+h} - \frac{3(3+h)}{3}}{3h(3+h)}$

Simplify: $= \lim_{h \rightarrow 0} \frac{3 - (3+h)}{3h(3+h)}$

$$= \lim_{h \rightarrow 0} \frac{-h}{3h(3+h)}$$

Cancel: $= \lim_{h \rightarrow 0} \frac{-1}{3(3+h)}$

Evaluate the limit: $= \frac{-1}{3(3+0)}$

$$= -\frac{1}{9}.$$

(c) $f(x) = \frac{2}{\sqrt{x}}$. Find $f'(4)$.

Definition of derivative at $x = 4$: $f'(4) = \lim_{h \rightarrow 0} \frac{f(4+h) - f(4)}{h}$

Use the function $f(x) = \frac{2}{\sqrt{x}}$: $= \lim_{h \rightarrow 0} \frac{\frac{2}{\sqrt{4+h}} - \frac{2}{\sqrt{4}}}{h}$

Simplify just a little: $= \lim_{h \rightarrow 0} \frac{\frac{2}{\sqrt{4+h}} - 1}{h}$

Clear mini-denominator (optional): $= \lim_{h \rightarrow 0} \frac{\frac{2}{\sqrt{4+h}} - 1}{h} \cdot \frac{\sqrt{4+h}}{\sqrt{4+h}}$

Distribute and simplify: $= \lim_{h \rightarrow 0} \frac{2 - \sqrt{4+h}}{h\sqrt{4+h}}$

Multiply by a conjugate: $= \lim_{h \rightarrow 0} \frac{(2 - \sqrt{4+h})}{h\sqrt{4+h}} \cdot \frac{(2 + \sqrt{4+h})}{(2 + \sqrt{4+h})}$

FOIL: $= \lim_{h \rightarrow 0} \frac{4 + 2\sqrt{4+h} - 2\sqrt{4+h} - (4+h)}{h\sqrt{4+h}(2 + \sqrt{4+h})}$

Simplify: $= \lim_{h \rightarrow 0} \frac{-h}{h\sqrt{4+h}(2 + \sqrt{4+h})}$

Cancel: $= \lim_{h \rightarrow 0} \frac{-1}{\sqrt{4+h}(2 + \sqrt{4+h})}$

Evaluate the limit: $= \frac{-1}{\sqrt{4+0}(2 + \sqrt{4+0})}$

$$= -\frac{1}{8}$$