Project 14–Antiderivatives, Indefinite Integrals, and Definite Integrals

1. Find the most general antiderivative for each of the following functions.

a) $\sin(x) + c$ $\qquad$ $\sin(3x) + c$ $\qquad$ $\sin(3x + 1) + c$

Solutions

$-\cos(x) + c$ $\qquad$ $-\frac{1}{3}\cos(3x) + c$ $\qquad$ $-\frac{1}{3}\cos(3x + 1) + c$

b) e^x $\qquad$ e^{2x} $\qquad$ e^{3x+1}

Solutions

$e^x + c$ $\qquad$ $\frac{1}{2}e^{2x} + c$ $\qquad$ $\frac{1}{3}e^{3x+1} + c$

c) $\frac{1}{x}$ $\qquad$ $\frac{1}{3x}$ $\qquad$ $\frac{1}{3x + 2}$

Solutions

$\ln(|x|) + c$ $\qquad$ $\frac{1}{3}\ln(|3x|) + c$ $\qquad$ $\frac{1}{3}\ln(|3x + 2|) + c$

d) $\sqrt{x}$ $\qquad$ $\sqrt{2x}$ $\qquad$ $\frac{1}{\sqrt{2x}}$

Solutions

$\frac{2}{3}x^{3/2} + c$ $\qquad$ $\frac{2}{6}(2x)^{3/2} + c$ $\qquad$ $\sqrt{2x} + c$

2. Evaluate each of the following indefinite integrals.

a) $\displaystyle\int \frac{1}{3x} dx$ $\qquad\qquad$ $\displaystyle\int \frac{2}{3x+5} dx$

Solutions

$\frac{1}{3}\ln(|3x|) + c$ $\qquad\qquad$ $\frac{2}{3}\ln(|3x+5|) + c$

b) $\displaystyle\int \frac{1}{e^x} dx$ $\qquad\qquad$ $\displaystyle\int \frac{1}{e^{3x+1}} dx$

Solutions

$-e^{-x} + c$ $\qquad\qquad$ $-\frac{1}{3}e^{-(3x+1)} + c$

c) $\displaystyle\int \frac{1}{\sqrt{x}} dx$ $\qquad\qquad$ $\displaystyle\int \frac{2}{\sqrt{3x+5}} dx$

Solutions

$2\sqrt{x} + c$ $\qquad\qquad$ $\frac{4}{3}\sqrt{3x+5} + c$

3. Find the area under each of the following curves over the indicated interval.

a) $f(x) = \dfrac{1}{3x+1}$ over $[1, 4]$

Solution

$$\int_1^4 \frac{1}{3x+1}dx = \frac{1}{3}\ln(3x+1)\Big|_1^4 = \frac{1}{3}\big(\ln(13) - \ln(4)\big)$$

b) $g(x) = \dfrac{1}{1+4x^2}$ over $[0, \frac{1}{2}]$

Solution

$$\int_0^{1/2} \frac{1}{1+4x^2}dx = \frac{1}{2}\arctan(2x)\Big|_0^{1/2} = \frac{1}{2}(\arctan(1) - \arctan(0)) = \frac{\pi}{8}$$

c) $\dfrac{1}{e^{2x+1}}$ over $[0, 2]$

Solution

$$\int_0^2 \frac{1}{e^{2x+1}}dx = -\frac{1}{2e^{2x+1}}\Big|_0^2 = -\frac{1}{2}\big(e^{-5} - e^{-1}\big) = \frac{1}{2}\big(\frac{1}{e} - \frac{1}{e^5}\big)$$

4. Find the area enclosed by the graph of each of the following curves in the first quadrant.

a) $f(x) = \sqrt{4-x}$

Solution

$$\text{Area} = \int_0^4 \sqrt{4-x}\ dx = -\frac{2}{3}(4-x)^{3/2}\Big|_0^4 = -\frac{2}{3}\left((4-4)^{3/2} - (4-0)^{3/2}\right) = -\frac{2}{3}\left(0 - 4^{3/2}\right) = \frac{16}{3}$$

b) $f(x) = \sqrt[3]{8-x}$

Solution

$$\text{Area} = \int_0^8 \sqrt[3]{8-x}\ dx = -\frac{3}{4}(8-x)^{4/3}\Big|_0^8 = -\frac{3}{4}\left((8-8)^{4/3} - (8-0)^{4/3}\right) = -\frac{3}{4}\left(0 - 8^{4/3}\right) = 12$$

c) $f(x) = -x^2 + 9$

Solution

$$\text{Area} = \int_0^3 -x^2 + 9\ dx = -\frac{1}{3}x^3 + 9x\Big|_0^3 = -\frac{1}{3}3^3 + 9 \times 3 = 18$$

d) $f(x) = \sqrt{9-x^2}$

Solution

$$\text{Area} = \frac{9}{4}\pi$$

5. Find the area enclosed by each of the curves and above the x-axis.

a) $y = -x^2 + x + 2$

Solution

$$\text{Area} \;=\; \int_{-1}^{2} -x^2 + x + 2 \; dx = -\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x\Big|_{-1}^{2} = \frac{9}{2}$$

b) $y = x^3 - 6x^2 + 8x$

Solution

$$\text{Area} \;=\; \int_{0}^{2} x^3 - 6x^2 + 8x \; dx = \frac{1}{4}x^4 - 2x^3 + 4x^2\Big|_{0}^{2} = 4$$