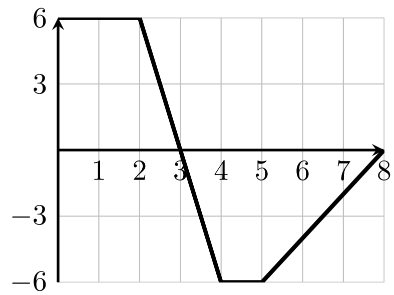
1. Review and Warm-up: The graph of f is shown below. Calculate exactly each of the definite integrals that follow.



(a) $\int_0^2 f(x)\,dx =$ **Solution:** 12

(b) $\int_0^3 f(x)\,dx =$ **Solution:** 15

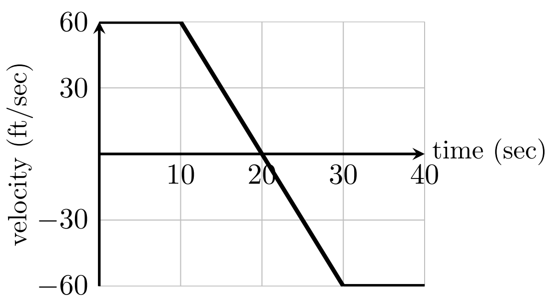
(c) $\int_4^5 f(x)\,dx =$ **Solution:** -6

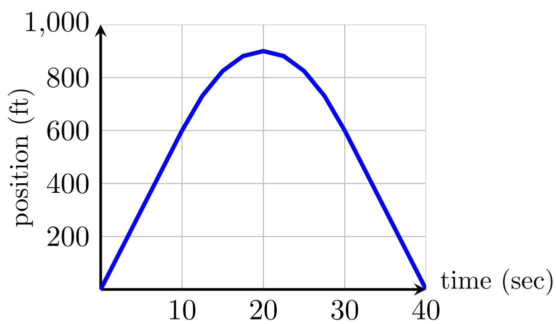
(d) $\int_5^8 f(x)\,dx =$ **Solution:** -9

(e) $\int_2^4 f(x)\,dx =$ **Solution:** 0

(f) $\int_0^8 f(x)\,dx =$ **Solution:** -3

2. Let $s(t)$ be the position, in feet, of a car along a straight east/west highway at time t seconds. Positive values of s indicate eastward displacement of the car from home, and negative values indicate westward displacement. At $t = 0$ the car is at home. Let $v(t)$ represent the velocity of this same car, in feet per second, at time t seconds (see graph below).





(a) Write a definite integral representing each of the following:

$s(10) =$ **Solution:** $\int_0^{10} v(t)\,dt$

$s(30) =$ **Solution:** $\int_0^{30} v(t)\,dt$

$s(t) =$ **Solution:** $\int_0^{t} v(u)\,du$

Now use these integrals and the velocity graph to help you fill in the chart below:

t	0	10	20	30	40
$s(t)$	0	600	900	600	0

(b) Use these values to help you plot the position function.

(c) Fill in the chart below:

Definite integral of velocity	Change in position
$\int_0^{10} v(t)dt = 600$	$s(10) - s(0) = 600 - 0$
$\int_{10}^{20} v(t)dt = 300$	$s(20) - s(10) = 900 - 600$
$\int_0^{40} v(t)dt = 0$	$s(40) - s(0) = 0 - 0$

(d) Why do these two columns give the same answers?
Solution: The definite integral of a velocity function with respect to time represents a change in position over the same interval of time.

3. The following data is from the U.S. Bureau of Economic Analysis. It shows the rate of change $r(t)$ (in dollars per month) of per capita personal income, where t is the number of months after January 1, 2012.

t (months)	0	2	4	6	8	10	12
$r(t)$ (dollars per month)	154	17	10	79	278	-432	290

Use left-hand Riemann sums to estimate the total change in personal income during 2012.
Solution:

$$\sum_{i=1}^{n} r(a + i\Delta t) \cdot \Delta t \quad \text{where, in this case, } a = 0 \text{ and } \Delta t = 2$$

$$\begin{aligned}
&= r(0) \cdot 2 + r(2) \cdot 2 + r(4) \cdot 2 + r(6) \cdot 2 + r(8) \cdot 2 + r(10) \cdot 2 \\
&= 2\left(r(0) + r(2) + r(4) + r(6) + r(8) + r(10)\right) \\
&= 2\left(154 + 17 + 10 + 79 + 278 - 432\right) \\
&= 212
\end{aligned}$$

4. A can of soda is put into a refrigerator to cool. The temperature of the soda is given by $F(t)$. The **rate** at which the temperature of the soda is changing is given by

$$F'(t) = f(t) = -25e^{-2t} \text{ (in degrees Fahrenheit per hour)}$$

(a) Find the rate at which the soda is cooling after 0, 1, and 2 hours. Then use this information to estimate the temperature of the soda after 3 hours if the soda was $60°F$ when it was placed in the refrigerator.
Solution: We can use another left-hand Riemann sum with $\Delta t = 1$ hr to find the *change* in temperature.

$$\begin{aligned}
&= f(0) \cdot 1 + f(1) \cdot 1 + f(2) \cdot 1 \\
&= -25e^{-2\cdot 0} + -25e^{-2\cdot 1} + -25e^{-2\cdot 2} \\
&\approx -28.84
\end{aligned}$$

If the temperature changed by $\approx -28.84°F$ over three hours, and started at $60°F$, then the final temperature is $\approx 31.16°F$.

(b) Now we will find the exact temperature of the soda after three hours have passed.

i. Find an antiderivative of $f(t)$, that is, find a function $F(t)$ such that $F'(t) = f(t) = -25e^{-2t}$.
(Hint: take a couple of derivatives of $f(t)$ and try to find a pattern.)

Solution:

$$f'(t) = 50e^{-2t} \text{ and } f''(t) = -100e^{-2t}$$

It looks like we gain a factor of -2 for every derivative we take, so to find our anti-derivative we should divide by -2. Then any function of the form $F(t) = \frac{-25}{-2}e^{-2t} + C = 12.5e^{-2t} + C$ will be an antiderivative of $f(x)$. Since the question only asked for one such function, we'll use $F(t) = 12.5e^{-2t}$.

ii. The Fundamental Theorem of Calculus (the Evaluation Theorem) tells us that $\int_a^b f(t)\,dt = F(b) - F(a)$. Use this theorem and the function you found in the last step to find the temperature of the soda after 3 hours have passed.

Solution: Evaluating the integral gives us:

$$\int_0^3 f(t)\,dt = F(3) - F(0) = 12.5e^{-6} - 12.5$$

We know from the FTC that

$$\int_0^3 f(t)\,dt = F(3) - F(0)$$

Substituting the evaluated integral and the given initial temperature $F(0) = 60$, we have:

$$12.5e^{-6} - 12.5 = F(3) - 60$$

Solving for F(3)

$$F(3) \approx 47.531°F$$

(c) Why do you think your estimate in part (a) is so far off?

Solution: $f(t)$ changes very quickly between 0 and 3. We should have used a smaller Δt

5. (a) Write an integral which represents the area between $f(x) = x^4$ and the x-axis, between $x = 0$ and $x = 2$.

   Solution:
   $$\int_0^2 x^4 \, dx$$

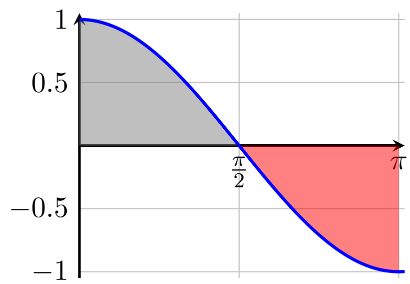
   (b) Evaluate this integral using the Fundamental Theorem of Calculus (the Evaluation Theorem).

   Solution: Any function of the form $\frac{x^5}{5} + C$ is an antiderivative of $f(x) = x^4$, thus $F(x) = \frac{x^5}{5}$ is an example of an antiderivative of $f(x) = x^4$, and so
   $$\int_0^2 x^4 \, dx = F(2) - F(0) = \frac{2^5}{5} - \frac{0^5}{5} = 6.4$$

6. (a) Using the Fundamental Theorem of Calculus (as in the last problem), evaluate $\int_0^\pi \cos(x) \, dx$.

   Solution: $F(x) = \sin(x)$ is an antiderivative of $f(x) = \cos(x)$, and so
   $$\int_0^\pi \cos(x) \, dx = F(\pi) - F(0) = \sin(\pi) - \sin(0) = 0$$

   (b) Show the area represented by the integral in part (a) on the graph.



7. Evaluate $\int_0^1 \frac{1}{1+x^2} \, dx$ without using technology.

Solution: $F(x) = \arctan(x)$ is an antiderivative of $f(x) = \frac{1}{1+x^2}$, and so
$$\int_0^\pi \frac{1}{1+x^2} \, dx = F(1) - F(0) = \arctan(1) - \arctan(0) = \frac{\pi}{4}$$