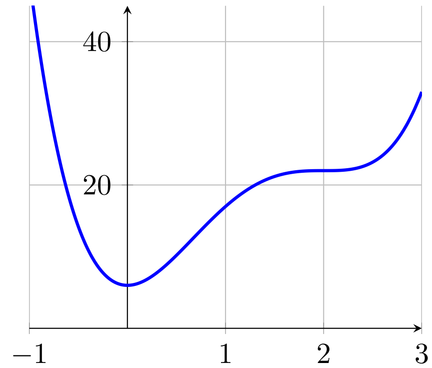
1. Use technology to graph $g(x) = 3x^4 - 16x^3 + 24x^2 + 6$.

   Solution:



   (a) Looking at the graph, where does it appear that $g(x)$ has relative minima and relative maxima (valleys and peaks)?

   Solution: There appears to be a low point (relative minimum) at $x = 0$.

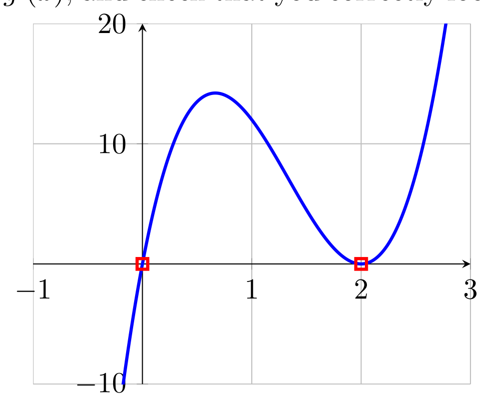
   (b) Looking at the graph, where does it appear that $g(x)$ has inflection points?

   Solution: g(x) appears to have inflection points at around $x = 0.5$ and $x = 2$.

   (c) Calculate $g'(x)$ and find where it is zero using algebra.
   Solution:

$$\begin{aligned} g'(x) &= 12x^3 - 48x^2 + 48x \\ 0 &= 12x^3 - 48x^2 + 48x \\ 0 &= 12x(x^2 - 4x + 4) \\ 0 &= 12x(x-2)(x-2) \\ \text{So } x &= 0 \text{ or } x = 2 \end{aligned}$$

   (d) Use technology to graph $g'(x)$, and check that you correctly found its zeroes. **Solution:**



(e) Now interpret the graph of $g'(x)$, explaining how it can be used to determine where $g(x)$ (the original function) has its relative minima and maxima.

Solution: Left of 0, $g'(x) < 0$, and right of 0, $g'(x) > 0$, therefore $g(x)$ has a relative minimum at $x = 0$.

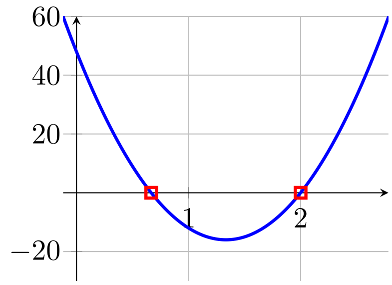
Left of 2, $g'(x) > 0$, right of 2, $g'(x) > 0$, therefore $g(x)$ does not have a relative low or high at $x = 2$.

(f) Calculate $g''(x)$ and find its zeroes.

Solution:

$$g''(x) = 36x^2 - 96x + 48$$
$$0 = 36x^2 - 96x + 48 = 12(3x^2 - 8x + 4)$$
$$0 = 12(x-2)(3x-2), \text{ so } x = 2, \frac{2}{3}$$

(g) Use technology to graph $g''(x)$, and check that you correctly found its zeroes.
Solution:



(h) Explain how you can use the graph of $g''(x)$ to determine the exact location of the inflection points of $g(x)$.

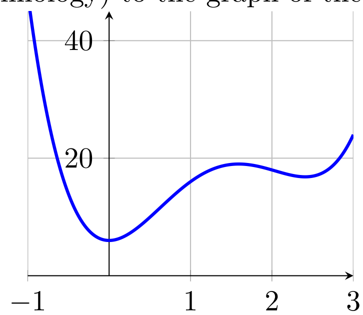
Solution: The sign of $g''(x)$ changes at the marked points. The zeroes of $g''(x)$ are close to where we guessed the inflection of points of $g(x)$ would be.

(i) Explain how you can use the graph of $g'(x)$ to determine the exact location of the inflection points of $g(x)$.

Solution: At $x = \frac{2}{3}$ $g'(x)$ has a local maximum. This means that $g'(x)$ switches from increasing to decreasing there. So the derivative of $g'(x)$, namely $g''(x)$, must switch from positive to negative there. This means that $g(x)$ switches from concave up to concave down at $x = \frac{2}{3}$, and thus has an inflection point. A similar argument can be made at $x = 2$, where $g'(x)$ has a local minimum. Bottom line: where the derivative has a local extreme point, the original function must have an inflection point

2. Now consider the function $f(x) = 3x^4 - 16x^3 + 23x^2 + 6$. Notice how similar its formula is to the function is in the previous problem.

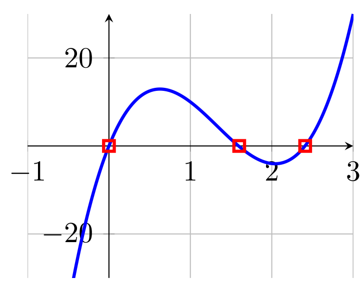
   (a) Compare its graph (using technology) to the graph of the previous function.
   Solution:



   (b) Looking at the graph, where does it appear that $f(x)$ has relative minima and relative maxima?

   Solution: Relative max at $x = 1.5$ and relative min at $x = 0$ and $x = 2.5$.

   (c) Calculate $f'(x)$ and use technology to graph it.
   Solution: $f'(x) = 12x^3 - 48x^2 + 46x$



   (d) Use the graph of $f'(x)$ to answer the question of where $f(x)$ has its relative maxima and minima. Explain.
   Solution: Left of 0, $f'(x) < 0$, and right of 0, $f'(x) > 0$, therefore $f(x)$ has a relative minimum at $x = 0$.
   Left of 1.5, $f'(x) > 0$, and right of 1.5, $f'(x) < 0$, therefore $f(x)$ has relative maximum at $x = 1.5$.
   Left of 2.5, $f'(x) < 0$, and right of 2.5, $f'(x) > 0$, therefore $f(x)$ has a relative minimum at $x = 2.5$.

   (e) What are the key differences between the graphs of $g(x)$ (from problem 1) and $f(x)$ (from this problem)? How are these differences reflected in the graphs of $f'(x)$ and $g'(x)$?
   Solution: $g(x)$ has only one local extreme point, and one stationary point. But in the graph of $f(x)$, that stationary point looks like it's been turned a little bit, so there's a now a local maximum and local minimum. There's a dip in the function, where water could pool if this was a roof. You can see this in the derivatives: $g'(x)$ has a zero (x-intercept) where the graph just touches the axis but does not change sign. The graph of $f'(x)$, however, dips slightly below the x-axis. $f(x)$ has a local extreme point at each of the places where the derivative crosses.