The goal of this project is to practice writing functions that model given situations. Each of the functions you create should depend on only one variable. You'll need this skill to solve applied optimization problems.

1. The product of two numbers n and m is 100, and n must be positive. (For example, $n = 20$ and $m =$__________ or $n = \frac{2}{3}$ and $m =$____________).
   (a) Write a function that gives their sum in terms of n. What is the domain of this function?

   (b) Graph that function using technology. What is its minimum value? Does it have a maximum value?

2. A rectangle with sides of length ℓ and width w has perimeter of 400 yards. (For example, if the length ℓ is 50 yards, then $w =$_________________ and the area A=__________________). Write a function that gives the area in terms of ℓ. What is the domain of this function?

3. A cone has a volume of 231 in^3, which is 1 gallon. (For example, if the height is $h = 5$ in then the radius is $r =$______________). Find a function that gives its radius in terms of its height. What is the domain of this function?

4. A rectangle has its base along the x-axis and its upper two vertices lie on the graph of the parabola $y = 12 - x^2$. Draw a picture of this. When the base of the rectangle has length 4, the area of the rectangle is ___________. Write a function that gives the area of the rectangle in terms of the length of its base. What is the domain of this function?

5. A rectangular box with a top and a square base has a volume of 1 L (1000 cm^3). For example, if the length of the base is $x = 5$ cm then the surface area is _______________. Find a function giving the surface area of the box in terms of x, the length of its base. What is the domain of this function?

6. A light-rail system carries 80000 passengers per day at a fare of \$2.25 per ride. For each 5-cent increase/decrease in fare, surveys predict ridership will drop/grow by 250 passengers. For example, if the fare is lowered to \$2.00 per ride, then the number of riders is ______________ and the revenue is ________________.

   (a) Find a function giving the revenue as a function of x, the number of 5-cent increases. (Hint: first find a formula for the number of riders as a function of x, and the cost per ride as a function of x.) What is the domain of this function?

   (b) The function you get should be a parabola which opens downward. Use calculus to find where the maximum value must occur. What should the fare be to maximize revenue?

7. A trucking company charges its clients \$30 for each hour of driving time, plus the cost of fuel. At a driving speed of v miles per hour, the trucks get a mileage of $10 - 0.07v$ miles per gallon. The fuel costs \$3.00 per gallon. Find a function giving the cost per mile to the client as a function of the speed the trucks drive. What is the domain of the function?

8. A cylinder is inscribed in a cone of radius 10 and height 20. Find a function for the volume of cylinder as a function of its radius. What is the domain of the function?